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DISSERTATION ABSTRACT

A SIMULATED ANNEALING APPROACH FOR THE COMPOSITE FACILITY LOCATION AND RESOURCE ALLOCATION PROBLEM: A STUDY OF STRATEGIC POSITIONING OF US AIR FORCE MUNITIONS

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The US Air Force faces the difficult decision of where to strategically pre-position munitions stocks in preparation for a variety of possible future wartime scenarios. This problem includes aspects of both the capacitated facility location problem and the resource allocation problem. The problem addressed is considered multi-objective in nature and cost minimization is balanced against minimizing the coverage distances that munitions must be transported to meet demands.

Typical solutions to the facility location problem and resource allocation problem do not take into account the constraints of the logistics environment. Therefore, this study incorporates transportation and facility costs, and uses actual geographic distances with adjustments made for available modes of transportation. Feasible solutions to the composite facility location and resource allocation problem are generated using a

simulated annealing algorithm that explores both inventory transfers and location transfers during the course of the search. Simulated annealing is a meta-heuristic technique analogous to the physical annealing of solids and has been successfully used in many operations research problems, but has not been applied to a problem of where to position strategic inventory.

The study uses an experimental design which tests the ability of the algorithm to provide improved solutions to the problem when using different search parameter values. Different inventory transfer sizes are used in the search in order to analyze the effects of repositioning inventory in larger packages than the typical transfer size of one unit. In addition, the search algorithm periodically redirects the search based on the best coverage solution found after a number of iterations. How often to accomplish this redirection is also an experimental factor of the study.

The results of the study indicate that munitions inventories can be pre-positioned to simultaneously improve both objectives of the problem in comparison to the existing initial solution. In addition, it is shown that the cost and coverage values achieved by the model depend on the configuration and size of the problem being solved. Also, the quality of the solutions is dependent on the combination of transfer size and reset frequency used by the algorithm. Improvement in the quality of solutions is evident when using the largest transfer size, and the most improved solutions are found when the transfer size is combined with the largest reset frequency. The results of the study also provide a means for analyzing which warehouse locations should be opened from the set of potential locations and what inventories quantities should be stocked at each location.

VITA

Major John E. Bell, son of Lonnie Richard and Martha (Pinnow) Bell, was born [redacted] Spokane, Washington. He graduated from Billings Senior High School in Billings Montana in 1986 and joined the U.S. Air Force shortly after graduation. Major Bell received a Bachelor of Science degree from the U.S. Air Force Academy in May, 1990, and was commissioned a regular officer in the United States Air Force. He has served as munitions maintenance officer in the United States Air Force for the last twelve years including assignments in Oklahoma, Texas, Ohio, and the country of Turkey. His duties have included management, staff and research positions including overseas deployments to Operations Desert Storm and Provide Comfort. In 1998, Major Bell became a Distinguished Graduate of the Air Force Institute of Technology in Dayton, Ohio, with a Master of Science degree in Logistics Management. After two years as a research analyst at Maxwell Air Force Base, Alabama, he entered Graduate School, Auburn University, in August 2000. Major Bell married [redacted] daughter of [redacted], on October 6, 1992, and they have three daughters: [redacted]

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Introduction

Purpose

The US Air Force faces a problem deciding where to locate munitions storage facilities and inventories in order to prepare for possible future wars. This Munitions Pre-Positioning problem has several possible objectives such as minimizing cost, maximizing demand coverage, or minimizing response time. This research develops a model to provide answers for how to best preposition US Air Force munitions inventories needed for future conflicts for a variety of demand scenarios. This model is a combination facility location model and inventory allocation model which is aimed at simultaneously determining where to locate facilities and how to position inventory quantities. It is the intent of the research to identify solutions that identify improved storage locations and munitions inventories stocking locations while taking into consideration the logistics constraints of potential future demands. The overall purpose of the research is to provide managers and strategic planners with an improved method for making decisions about facility location and inventory positioning problems.

Facility Location Problems

Mathematicians and other scientists have studied the optimal placement of facilities since at the 19th century. However, one of the most well-known and widely studied location problems was formulated in the early part of the 20th century by Alfred

Weber in an attempt to find the most cost-effective location for production based on the existing location of raw materials and customer demands. This problem, known as the "Weber Problem", attempts to find a single location that minimizes the distances from the selected facility site to the supply and demand locations. An extension of this problem, the p-median problem also seeks to find the optimal location for a fixed number of facilities (p) that minimizes the total distance to a set of selected locations. Models for solving this problem later became known as Location-Allocation models since they must simultaneously determine the location of several supply facilities, and determine how to allocate the demand locations to the supply locations. An extension of the p-median problem, the Facility Location Problem (Efroymson and Ray,1966; Spielberg, 1969; and others), seeks to minimize the total costs of a system by including the fixed and variable costs associated with locating and operating a distribution center instead of considering only the transportation costs related to distance. The FLP can be formulated with either uncapacitated or capacitated facility locations (UFLP/CFLP) (Daskin, 1995). Unlike the p-median problem, the Facility Location Problem (FLP) determines the number of supply locations (p) to select and provides them as a model output. Therefore, the number of locations (p) is produced from within the model and is said to be "endogenous". Endogenous selection allows the model to either close or open potential facilities in order to find the least cost solutions, instead of restricting the number of supply locations to an exogenous number (p) determined outside the model by the user. In addition, the FLP can be classified by whether or not a demand point may be supplied or serviced by more than one location. In the single-source version of the FLP, each demand is supplied from

one supply location and in the less frequent multiple-source version of the problem each demand may be supplied by more than one site.

Many other variations of the p-median problem and FLP have been formulated including the minimax problem which uses an objective function that minimizes the maximum distance of any demand point from the supply locations. In addition, covering problems have been developed that use a maximum distance constraint between the supply and demand locations. The two most cited covering problems are the Location Set Covering Problem (LSCP) (Toregas and Revelle, 1972) and the Maximum Location Covering Problem (MLCP)(Church and ReVelle, 1974). The LSCP attempts to minimize the number of facilities to cover a given demand and the MLCP aims to maximize the quantity of demand covered with a limited number of supply locations. A great deal of the research for both types of covering problems has been conducted for locating emergency service facilities and resources for fire departments and hospitals.

During the past forty years, much effort has been devoted to solving the p-median location allocation problem and the facility location problem. In addition, many added features have been included in location models including multiple objectives, multiple facility types, multiple time periods and consumer characteristics. Researchers have also studied several methods to more accurately determine the distances used to calculate feasible location decisions and have studied the errors that can result from aggregating demand data in location models. In addition, dynamic location allocation models have been formulated that take into consideration the effects of multiple time periods and stochastic supply and demand quantities resulting from uncertainty about the future. One

enhancement to location-allocation models has been the recognition that location decisions may be tied to equally important resource allocation decisions. Decision makers may not only be concerned with locating a set of facilities, but might also need to solve the underlying problem of how much capacity or inventory to locate at a facility once it is opened.

Resource Allocation Problems

The resource allocation problem is concerned primarily with allocating a scarce set of resources among competing activities. This problem has been studied primarily in reference to production planning and scheduling, as well as the placement of emergency services and strategic inventories at different locations. These problems are usually modeled as either maximal covering problems that attempt to serve all demand points or as minimax problems that attempt to minimize the maximum time or cost of meeting any demand. Resource allocation problems may be concerned with determining the amount of capacity to place at more than one location. For example, a manufacturer may have several manufacturing locations, but may only have a limited amount of specialized equipment or skilled labor to locate at these facilities. Resource allocation might also include the placement of inventories at storage locations in anticipation of future demand. For public organizations, the placement of strategic inventories is sometimes necessary to protect against the high costs of a stock-out. This is especially important when demand is highly uncertain or unknown. Resource allocation problems may also take a multi-item approach where different characteristics such as cost, size or value may be associated

with dissimilar resources. For example, a problem for the placement of strategic inventories might include a multi-item aspect to the problem where different values and costs are associated with the various types of inventories allocated to facility locations. Research on resource allocation (as it applies to inventory) has focused on the advantages of pooling inventory at centralized locations in order to meet varying customer demands and the inherent tradeoff between covering customer demand in a timely manner and minimizing overall system costs.

Combined Location and Resource Allocation Problems

A limited amount of research has been conducted on problems that combine location analysis with resource allocation; perhaps because formulating and solving these problems using linear or mixed integer programming is difficult due to the combinatorial nature of the problem. Therefore, the amount of research for such combined problems has been limited. Research on distribution system design has occasionally taken into account the costs or constraints associated with maintaining inventory levels at different locations. In addition, the placement of capacity resources such as vehicles or production equipment has been considered in previous location research. However, much of the research that considers combined location and resource allocation problems has been limited to uncapacitated location or single-item problems, and usually it does not take into account the difficulty associated with multi-item or even multi-objective problems. In addition, much of this work has not taken into account many real world logistics

constraints and has not included transportation characteristics needed to more accurately model the actual movement of material from supply to demand locations.

Transportation Considerations

Distances used for solving location models are typically calculated using simple mathematical techniques such as Euclidean or rectilinear computations. Although using such methods in location science reduces the difficulty of mathematical computations, it does not always capture the true nature and cost of transporting material between supply and demand locations in an actual transportation network. One improvement to a location model can be made by using actual geographic distances as calculated using the actual longitude and latitude coordinates for locations included in the model. Even then however, equal distance does not always represent equal time or cost in a mathematical model. For example, even though two supply locations may be equidistant from a demand location using a straight line distance computation, they may not represent an equal choice for serving the demand if they do not both possess equal transportation characteristics. The mode of transportation (rail, truck, air, or sea) between a supply location and the demand location can have a significant impact on the distances traveled and the eventual costs of supplying the demand. Such differences can make one supply location superior in its ability to meet the demand even though no difference was evident in the straight line distance computations. Therefore, the ability to model aspects of the transportation network such as different modes of transportation from supply to demand locations becomes a necessary aspect of the combined location-resource allocation

problem when the decision-maker needs to more accurately represent transportation and distances in a geographic region.

Strategic Positioning of Resources and Materials

Facility location problems continue to be an important part of strategic planning and decision making for logistics managers today. This is especially true for the positioning of facilities needed to store emergency response materials for war, natural disaster, or fire response. These decisions are characterized by uncertainty in demand, infrequent or low demand, short lead-times for response, and a high cost of not meeting demand. Due to these characteristics, such facility location problems must consider the strategic positioning of inventory or other resources among the facilities that are being positioned. The majority of the research in this area has been related to the problem of where to locate fire stations and how to position emergency response vehicles. However, other government and public service organizations face a similar problem for positioning consumable inventories of supplies needed for disaster or war response. Organization such as the Red Cross, the Federal Emergency Management Agency (FEMA), and the US military encounter the problem of simultaneously determining where to build facilities and determining how much inventory to pre-position at each location.

Munitions Pre-Positioning Problem

The United States Air Force must determine where to position war readiness material (WRM) to be used in future conflicts. This difficult problem is especially

troublesome for dangerous and cumbersome resources such as air-to-ground munitions used by attack aircraft in an air war. The nature of the problem has several interesting aspects. First, since there are many potential conflicts and operating plans, the lists of required WRM munitions to fulfill all potential operating plans far exceeds the actual stocks of munitions available for pre-positioning. Any set of demands used to plan the positioning of munitions must take into account possible Major Theater Wars as directed by the National Military Strategy and must also be flexible enough to response to unexpected small scale contingencies and conflicts that typically flare up around the world. One portion of the problem may therefore be thought of as a Resource Allocation problem where a limited amount of resources must be strategically allocated among a group of storage facilities to respond to a varying set of demand situations. Second, determining the location of inventory stocking locations and how to allocate demand to each location is an additional challenge. This second challenge is a location-allocation problem where the choice of location is restricted to a predetermined set of potential locations. These sites are typically at or near transportation hubs such as airfields, ports, or rail yards belonging to US allies and are constrained by the sites ability to safely store a fixed number of explosives as well as many other political considerations. In addition, this aspect of the problem may be constrained by the lack of funding to build additional storage facilities and is tied to a set of initial operating locations which were built during the Cold War. All of the potential and initial storage sites may additionally have unique fixed and variable costs for being opened, closed and operated depending on their location. Third, the global nature of military positioning must take into consideration the

diverse (and sometimes hostile) nature of the transportation networks around the world. In different parts of the world, the movement of munitions is: constrained to particular routes, utilizes different modes of transportation, and may transit several locations before reaching its final destination. Overall, an optimal solution to the Munitions Prepositioning problem should identify the best set of storage locations with the best mix of munitions inventories that considers transportation and other logistics constraints in satisfying a set of potential demands for future conflicts.

Multiple-Sourcing. During a conflict, it is natural to assume that munitions currently positioned at a combat location are always used first in meeting the demands of a conflict. However, if the demand is larger than supply at the current combat location then munitions will have to be shipped from other locations to fill the unmet demand. This logic establishes that the Munitions Pre-positioning model should use multi-sourcing and establishes a heuristic rule that shortages in demand quantities should be sourced in order from the nearest supply location to the most distant one. This rule is used in this research to first try to satisfy demand from quantities already stored at a demand location and then proceeds to the next nearest facility in an iterative process until all of the demand for the conflict is met. Use of this rule also implies ~~that~~ that no shortages are allowed. This is a reasonable assumption since in wartime Air Force operators are rarely satisfied with the concept of not supplying a quantity of munitions for a combat mission, and munitions will be transported in from the farthest geographic points on the globe to meet a combat requirement if necessary. However, a limitation to this model is the assumption that no conflict will exceed the available global quantities. This seems

reasonable since by the time all global quantities for a particular weapon type are consumed, the Air Force will have either started a new production line for the weapon or will have approved a reasonable substitution of an item that can closely match the requirements of the weapon whose inventory has been depleted.

Since required munitions may be shipped to the wartime demand location from many locations, the calculation of coverage values is not as straight forward as seen in previous research. In order to calculate the coverage objective in this study, an improved measure, the maximum average coverage value is created to take into account the fact that munitions may be multiple sourced and arriving at a demand location from several supply locations. This value is simply the maximum average distance that a group of munitions for an individual weapon type are required to travel to their particular demand location. The calculation of the maximum average cover value is described in detail in the Methodology chapter.

Cost Considerations. The global nature of the Munitions Pre-Positioning problem lends itself to using a variety of costs for facilities and transportation. For example, any model of the problem must acknowledge opening costs for any new munitions storage locations based on the amount of munitions to be stored at the location and the resulting number of storage warehouses that must be constructed. Similarly, closing costs of a munitions site must be taken into account to accurately reflect the decision to abandon a current location. Additionally, since the Munitions Pre-Positioning Problem covers a number of geographic locations including dozens of different foreign countries and even different continents, the costs for transporting materials depends

greatly on the region of the world and the available transportation modes. The costs of shipments being considered in the problem must be differentiated by both the mode of transportation and regional differences in transportation costs. This research uses the location and region of the shipping origin in order to determine total transportation costs for a specific shipment.

Transportation Restrictions. The availability of transportation resources and the actual distances traveled from source locations to destination sites in an actual conflict can vary greatly depending on a number of wartime factors. It is this variation that makes the use of straight-line distance computations inaccurate and misleading for a facility location model. The only mode of transportation that may arguably use straight-line distance is air transportation. Of course, this assumes that there are not any over-flight restrictions necessitating that neutral or enemy countries not be flown over. The use of straight-line distances actually implies the use of air cargo transportation which is a faulty assumption for the movement of mass quantities of heavy and hard to handle munitions. For example, during Operation Desert Storm only 26,000 short tons (8%) of the required 326,000 short tons of Air Force munitions were moved to Southeast Asia by airlift (AFMC, 1992). The remainder were all shipped to the conflict using surface transportation. Similar results were seen in the Kosovo conflict from February to June 1999 when over 460 railcars, seven coaster ships, and 1042 transport trucks were used to ship over 9,000,000 lbs. net explosive weight of munitions to combat locations in Southern Europe (USAFE, 1999). The dominance of surface transportation modes in comparison to air transportation makes it necessary to adjust of distances used by models

to more realistically model actual distances traveled during a conflict. This research places a limit on the amount of munitions moved by air to approximately ten percent and in addition adjusts straight-line distance computations using a circuitry routing factor as recommended by Bramel and Simchi-Levi(1997). This adjustment to straight-line distances is used in order to simulate surface (road, rail and sea) transportation distances. The actual routing factors for air and surface transportation calculations are listed in the Methodology chapter.

Conflict Size and Munitions Demand. The location and size of conflicts is subject to a high level of uncertainty and the quantity of munitions demanded can vary greatly from one conflict to the next. Military planners must pre-position munitions inventories in preparation for a wide array of conflicts from small operations lasting only weeks to major wars lasting several months. National Military Strategy dictates that the US Air Force will be prepared to respond for the possibility of at least two overlapping Major Theater Wars (MTWs) defined in military planning documents and operational plans (QDR,2001). For example, a possible conflict in the Korean Peninsula or the Persian Gulf Region would most likely constitute a significant military effort, involve many nations and be classified as an MTW. Predetermined military operating plans for these MTWs identify possible munitions quantities and consumption rates in the event the MTW occurs. However, many conflicts entered into by the US military such as the conflicts in Kosovo and Afghanistan are not defined in any pre-developed operating plan. Therefore, munitions pre-positioning decisions must also take into account unpredicted conflicts commonly described by military strategists as Small Scale Contingencies (SSC).

Therefore, an additional aspect of the Munitions Pre-positioning problem is the wide range of possible conflicts and demand sizes that a robust pre-positioning of munitions must be able to service. This research tests the combined location and resource allocation model on a range of problems taking into account MTW, SSC and a mixture of possible conflict scenarios at locations around the globe. This approach is in line with current military strategy which stresses that the US military has not abandoned its previous commitments to the two MTW construct, but instead "is changing the concept altogether by planning for victory across the spectrum of possible conflict" (QDR,2001). The occurrence of past conflicts, demand data from previous Air Force research and current political conditions are all taken into consideration in order to provide a reasonable demand set for studying the munitions-pre-positioning problem. However, the problem sets are in no way meant to represent all possible conflict scenarios, nor are they meant to represent the US Air Force's forecast of future events.

Multiple Objective Nature of the Problem

Several different criteria may be used as the objective for solving location and resource allocation problems. One of the most common objectives for a location model is to minimize the total costs of the system. The costs included in such a model usually include the cost associated with opening or closing facilities, the cost of operating facilities and the cost of transporting inventory between demand and supply locations. However, cost is not always the most important or only objective for a decision maker when solving a location and resource allocation problem. Fulfilling demand may be an

equally important objective for an operation that is extremely sensitive to stock-outs. For a military or emergency response organization, covering all of the demand on the system may be critical to mission success for winning a war and can be tied to life or death situations similar to those faced by fire and police departments. In these situations, costs are not the only objective; however, they cannot be forgotten and must be weighed against the ability of the system to meet all demands. For such a problem, a multi-objective approach for the location-resource allocation problem is a logical technique for finding solutions that not only attain reasonable cost levels, but also insure high levels of demand coverage. For analyzing the Munitions Pre-positioning problem in this research, these two competing objectives are necessary to ensure that on one hand the US military does not waste limited defense dollars appropriated by Congress, and on the other hand is able to provide overseas positioning necessary to quickly bring munitions to major combat operations. However, the multi-objective approach is difficult to solve and adds an additional level of complexity to the objective function of the problem. As the model tries to select new feasible solutions to improve the value of one objective, the value of the second objective may worsen and thereby undermine the improvement found in the first objective. Therefore, it is important to identify and analyze tradeoffs between the competing objectives.

One method for analyzing solutions for a dual-objective model is to map an efficient frontier or Pareto Front to be able to visually analyze the model results in order to determine the relative tradeoff between the different objectives of the model. For example, plotting the feasible solutions that attain a minimal cost for different levels of

coverage actually presents the decision maker with a choice of feasible solutions instead of relying on a single optimal output of a model. This technique allows the researcher to analyze the sensitivity of the model to changes in a constraint or input variable and provides flexibility to the decision maker for making the final location and allocation decisions. Having this flexibility is especially important since it is almost impossible to model all of the real-world constraints for a problem and a single optimal solution created by a mathematical model may not be implemental due to physical or political constraints not captured in the model.

Modern Search Heuristics

Several methods have been used to solve location allocation and resource allocation problems. Linear programming, Mixed Integer Programming and Non-Linear Programming have each been used to solve location problems of limited size. However, finding an exact solution to combinatorial optimization problems becomes increasingly difficult and subsequently impossible as the size of the problem increases. Location problems such as the Weber problem, p-Median problem and their extensions are described as Non-Deterministic Polynomial Hard (NP-Hard) in that no known exact algorithm is available to solve such problems in every instance. Therefore, heuristic techniques are commonly used to solve large location problems and much of the classic work done to solve simple location problems has been done using local search heuristics. Unfortunately, these techniques do not necessarily provide a global optimal solution and the solution process can become trapped at a local minimum. Researchers have therefore

developed a group of global search heuristics such as Genetic Algorithms, Tabu Search, Neural Networks, Ant Colony Optimization and Simulated Annealing to improve the search of the feasible set of solutions with the intent to find a near optimal result for large combinatorial optimization problems. Each of these techniques has gained merit for solving optimization problems. However, this research focuses primarily on the use of Simulated Annealing as a tool for solving large location-resource allocation problems. The technique of Simulated Annealing has provided results for other similar location and resource allocation problems, has a well established ability to explore feasible solution spaces, and does require the computational difficulty required be several of the other search heuristics. Therefore, this technique offers itself as a natural choice for exploring solutions to a new problem such as the Munitions Pre-positioning Problem.

Description of the Research

The research uses simulated annealing to find near optimal solutions to the Munitions Pre-positioning Problem which simultaneously consider the objectives of least cost and maximum average coverage. In addition, the research considers the positioning of variety of munitions types, uses constraints on the physical capacity of storage warehouses and allows shipments to be sourced from multiple shipping locations if necessary. The research takes into consideration realistic demands for US Air Force munitions based on possible Major Theater Wars and possible Small Scale Contingencies. The heuristic solutions generated by the research provide insight about the best location for current inventories and help evaluate the tradeoffs for re-positioning

inventory to new locations. In addition, the model considers the possible construction and use of new munitions sites and evaluates the effect of modes of transportation on the location and inventory positioning decisions. A more detailed description of the research can be found in the methodology section of this dissertation.

Intent of Research

The purpose of this research is to provide managers and strategic planners with an improved method for simultaneously making decisions about facility location and inventory positioning problems. The objective of the method may be to minimize the cost of the system, provide maximum coverage of demand or a multi-objective combination of these two objectives. Some organizations which face this problem may be primarily concerned with minimizing costs, while others may be more concerned with meeting each demand in order to ensure mission accomplishment. However, it is not the intent of this research to dictate which strategy is best, but instead, it is to provide the decision maker with a flexible methodology to address specific location-resource allocation problems such as the Munitions Pre-positioning Problem. The methodology explores different search criteria, heuristics and model parameters in order to identify the best search techniques for a problem of this nature. In addition, it is hoped that decision makers will further begin to understand the tradeoffs inherent to selecting different solutions to the problem and be able to use the methodology to analyze different planning scenarios and demand sets in order to understand the robustness or vulnerabilities of any location and inventory positioning decision.

Literature Review

In order to understand the composite location-resource allocation problem, it is necessary to first review the relevant literature for a variety of facility location problems and resource allocation problems.

Facility Location Problems

Since location problems have been studied in great detail for many decades, it is not surprising and worth noting that several comprehensive reviews of the literature pertaining to location analysis are available. Aikens (1985), Brandeau and Chieu (1989), and others have written extensive reviews of the location literature and describe the various characteristics of the problems studied and optimization techniques used in the research. In addition, several texts describing location analysis problems and methods have been developed by Ghosh and Rushton (1987), Mirchandi and Francis (1990), Drezner (1995), Daskin (1995) and others.

The Weber and P-Median Problems. Probably the most well-known location problem was formulated in the early part of the 20th century by German researcher, Alfred Weber. Weber, whose work was later translated into English, attempts to find the most efficient location for production between a group of raw materials and a set of demands (Friedrich, 1929). For many years, it was thought that this problem, the "Weber

Problem" could not be solved analytically (Ghosh and Rushton, 1987). For several decades, only limited success was achieved in solving small geometric problems containing three destination points and equal weighting of costs (the Steiner Problem). However, in the early 1960s, several researchers were able to develop solution methods to the Weber Problem using heuristic methods for larger problems and unequal weighting.

One of the first successful attempts to find solutions to the Weber problem is presented by Kuhn and Kuenne (1962). They generalize the Weber problem to instances that contain more than three destination locations and unequal transportation costs. Using an iterative algorithm they are able to locate the most "efficient" supply point on a continuous plane for problems with as many as 24 destination points. Their algorithm uses a minimum sum of squares or center of gravity approach that maps one trial solution to the next until the change between iterations becomes negligible. They demonstrate this technique for locating a population center by using Euclidean coordinates for cities in Russia and using population data to weight the demands for each city.

In their work, Kuehn and Hamburger (1963) employ heuristic methods to find near optimal solutions to the Weber problem as it applies to a large scale distribution system in the United States. The method consists of a two-stage heuristic with the objective of finding the minimum cost. The first stage starts with no warehouses and incrementally adds warehouses to the location that provides the largest cost reduction until no further cost reducing additions are available. Then the second stage employs a local search routine that provides further improvements to the solution found in Stage

One by either eliminating unnecessary warehouses or exchanging them with another possible location. In this manner, Kuehn and Hamburger were able to show how significant cost savings could be achieved in twelve sample problems that employ different initial manufacturing locations, and different cost structures. This work argues the merit of using heuristic methods to solve real world business problems due to their flexibility and ability to reduce solution time in order to solve large scale problems. For many years, the methods and sample problems provided by Kuehn and Hamburger have been used as a baseline of comparison for additional research for solving the Weber problem and related location problems.

A next important step was accomplished by Cooper (1963) who formulated the "p-median problem" which seeks to find the optimal location for a fixed number of source locations, "p" that minimizes the cumulative distance to a set of demands destinations. Since there are multiple locations, this problem is a location-allocation problem where the allocation of demand destinations to sources must be simultaneously determined. Cooper shows the exact mathematical methods for allocating demand to possible source locations; however he shows that due to the combinatorial nature of the problem finding exact solutions to larger problems is restricted by computational ability and cost. Therefore, he develops an approximate heuristic method that iteratively seeks to find the least cost solution by considering all of the possible allocations for each combination of supply locations. This method however is restricted to only considering the fixed destination sites as possible supply locations and therefore will only find a near optimal solution. However, in a group of eight small sample problems Cooper is able to

show that the approximate method has the ability to find near optimal solutions. In further research, Cooper (1964) analyzes three additional heuristic methods for solving the p-median problem and determines that a heuristic method that employs random selection of supply locations from the destination set is equally able to find good solutions to the p-median problem, but with much less computational time expended.

The p-median problem was then extended to finding optimal locations on a discrete network instead of in continuous space. These discrete network p-median problems are also sometimes called site selection problems. Hakimi (1964) provides the mathematical methods for finding the median of a graph in order to find the best discrete location for a switching center in a communication network when the location is restricted to one of the nodes on the graph. In addition, Hakimi also provides a method for determining the absolute center of a graph in order to determine the location of a police station with the desired outcome of minimizing the maximum distance the police would have to travel in order to respond to an accident. An additional warehouse location model is formulated by Maranzana (1964) whose partitioning algorithm is used to find the location of several supply locations on a transportation network while considering transportation costs and the shortest transportation paths along the network. The heuristic algorithm used by Maranzana can not guarantee convergence on an optimal solution; however, the method was found to be quite successful in finding a near optimal solution when using different initial solutions in repeated application of the algorithm.

Additional research by Tietz and Bart (1968) re-examines the network version of the p-median problem formulated by Hakimi and Maranzana, the selection of (p)

uncapacitated source locations from (n) destinations with equal demands and located on the nodes of a network. In their analysis, the researchers point out that the partition algorithm of Maranzana can at times be erratic and subject to high error variance. Therefore, they develop a new Vertex Substitution Algorithm which uses an iterative exchange process to select new source vertexes to improve the objective function and find a good approximation of the general median solution. In experimental experience, this substitution algorithm outperforms the partition algorithm. The Tietz and Bart algorithm provides a baseline for comparison for solution techniques to the general p-median problem on a graph.

ReVelle and Swain (1970) provide a warehouse site selection model which can be considered a further advancement in the evolution of solutions for the p-median problem on a network. There work makes use of linear programming to find the bounds for optimally locating facilities on a road network. Their solutions are then attained by using a Branch and Bound technique to find the optimal integer solution. The technique is successfully applied by ReVelle and Swain to problems with up to thirty locations and six warehouse sites (p). Further work, using these techniques belongs to Kuenne and Soland (1972) who also use an exact Branch and Bound method to find solutions to small p-median problems. These techniques and other which attempt to use exact algorithms to solve the p-median problem are limited by the size of the problem that can be solved. This is due to the fact that the p-median problem on a network has been shown using complexity theory to be NP-Hard (Kariv and Hakimi, 1979). This condition means that the time to solve the problem grows exponentially with the number of sources (p) being

located. Therefore, the use of heuristic methods continues to be a necessary and acceptable method for finding solutions to the p-median problem and its many variations.

Continued significant work for solutions to the P-Median Problem are presented by Khumawala (1973) who creates Delta and Omega heuristics to iteratively open and close facilities based on which potential location can most improve the objective function of a distance constrained p-median problem. In addition, , Hillsman and Rushton (1975) and Hillsman (1980) suggest additional heuristic algorithms for solving larger p-median problems in the many versions of their ALLOC computer program for solving p-median location problems including the Hillsman-Rushton algorithm and the Trade-Off algorithm. However, much of the more recent work for solving the p-median problem has used Lagrangian relaxation approaches as demonstrated by Daskin (1995). This method appears to consistently provide superior results in comparison to the earlier developed heuristic methods for small and medium sized problems. However, as the problem size grows the heuristic algorithms appear to consistently provide near optimal solutions close to the best value attained by Lagrangian relaxation, and they continue to offer the needed savings in time and computational difficulty required for larger problems. Further advancements in heuristic methods for the p-median problem including projection (Bongartz et al., 1994), Tabu Search (Brimberg and Mladenovic, 1996) and Variable Neigborhood Search have been compared by Brimberg et al (2000) who have analyzed some of the largest problems to date. Brimberg concludes that, “no one method is best in all cases and that the variation of strategies is limitless in terms of shaking (global search), local search and parameter setting. However, Brimberg’s results

indicate that relocation-based or interchange methods appear to be more efficient than other methods and Variable Neighborhood Search can effectively be used to obtain superior solutions.

Continued research on the p-median problem in the last several decades has added considerably to the difficulty of the problem by including multiple commodities, dissimilar facility types, multiple objectives and many other variations to the original problem. One major change is the addition of cost data to the problem which thereby transforms the problem into a facility location problem with the objective of finding the location of an unspecified number of facilities that minimizes cost for the entire system.

Uncapacitated and Capacitated Facility Location Problems (UFLP/CFLP). When the p-median model is adjusted to endogenously determine the number of locations by considering the fixed costs associated with opening a plant, the resulting problem is called the Facility Location Problem and can be formulated with either capacitated (CFLP) or uncapacitated locations (UFLP). One of the first instances of the UFLP is presented by Efroymson and Ray (1966) who formulate an integer programming problem for their "Plant Location Problem" which minimizes total costs consisting of both the transportation costs between each plant and its customers, and the fixed costs associated with building each plant. They find solutions to the problem using Branch and Bound methods and constrain their solutions to the single-source case where each demand location can only be supplied from one plant. Their work provides simplifications to the Branch and Bound algorithm that reduce the number of branches that have to be considered, thereby reducing the time necessary to find solutions for problems with up to

fifty potential plant locations and two hundred customers. In continued work on the UFLP, Speilberg (1969) presents computational experience on a variety of plant location problems with different costs and concludes that algorithms that are adaptive or closely matched to the problem data are the most effective for solving the plant location problem. He also considers the capacitated version of the problem and provides suggestions for solving such a problem.

The first algorithms for solving the CFLP are published in the literature by Davis and Ray (1969) and Sa (1969). In addition, one of the first researchers to formulate and solve the capacitated version of the simple facility location problem as it applies to warehouse distribution systems is Elson (1972). He finds successful solutions using a matrix generator program that finds optimal solutions to the mixed integer programming formulation of the problem. Later however, Akinc and Khumawala (1977) outline a more efficient solution method for the CFLP when they adapt the branch and bound methods of Khumawala to the problem. Their methods include developing hybrid node selection rules and testing of a variety of branch selection rules for finding improved solutions to the problem within a reasonable time (usually less than a minute). They are able to find solutions to twelve problem sets including problems originally developed by Kuehn and Hamburger (1963). These problems range in size from ten to twenty-five potential warehouse locations and twenty to fifty customer locations. In addition, their formulation of the CFLP does not necessarily restrict the model to the single-source case as more than warehouse may supply a portion of the demand for a particular customer. The work of Akinc and Khumawala is still considered some of the most important work

on the CFLP and is used today for comparison for new solution techniques for the problem. Additional work on the CFLP using branch and bound techniques is carried out by Neebe and Rau (83) and others.

A majority of the original work on the CFLP is based on linear programming and branch and bound solution methods. However, other research uses Lagrangian Relaxation as a method for finding solutions to the CFLP. This technique is first applied by Geoffrion and McBride (1978) who show that Lagrangian relaxation is much faster in a set of test problems in comparison to a branch and bound method. The problems consist of up to twenty four potential supply locations and over one hundred customer locations. Additionally, Lagrangian techniques are again effectively applied by Klincewicz and Luss (86) who use a Lagrangian heuristic method to find relatively fast and efficient solutions to the problem. Using this technique, they relax the capacity constraint to the problem and determine a lower bound using a dual ascent algorithm on the uncapacitated version of the problem. Initial solutions for their Lagrangian heuristic are generated using a simple add heuristic and a post-processing heuristic is used to make final adjustments to customer allocations of the final solution. Klincewicz and Luss apply their Lagrangian heuristic to the test problems of Kuehn and Hamburger (1963) and compare the results to an Add Heuristic for solving the problem. The results show that that the Lagrangian method can satisfactorily be applied to solving the CFLP and is superior to the Add Heuristic algorithm. Additional research on the applying Lagrangian relaxation to the CFLP is done by Pirkul (1987). This work provides improved methods for attaining the lower bounds for the problem and uses a heuristic method to find

generally superior results compared to those from previous research. Pirkul's method is described by Bramel and Simchi-Levi (1997) who apply the technique to a set of single sourced CFLP with up to one hundred potential warehouse sites and one hundred retail locations. The average error for the solutions compared to the lower bound for the problems is less than 2% in all instances.

Additional work on the CFLP occurs throughout the literature in the last decade, however, typically this work involves additional aspects of the original problem including multiple periods, multiple objectives or multiple items. Many of the problems described in the following sections are extensions of the CFLP or use the CFLP as a baseline to formulate more complicated problems. An overview of literature on distribution system design using the CFLP and associated algorithms is provided by Geoffrion and Powers (1995). Other extensions of the CFLP are described in the following sections.

Multi-Item Facility Location Problems

The literature contains many instances of the p-median and facility location problem that consider multiple items with dissimilar logistics characteristics such as size, cost, and demand preference. The first model to consider multiple items is Warszawski (1973) which outlines both a branch and bound and heuristic method for solving a multi-item facility location problem. Warszawski, however, believes the branch and bound method to be too computationally inefficient and concentrates on the testing of his heuristic method to solve the multiple-item uncapacitated FLP. Geoffrion and Graves (1974) formulate and solve multiple-item instances of the p-median and facility location

problems using an exact algorithmic method called Benders Decomposition. Other multi-item research by Neebe and Khumawala (1981) readdress the multi-item facility location problem formulated by Warszawski and provides an improved branch and bound method for its solution. Their algorithm uses an improved node selection rule which is a combination of two previously used rules, least lower bound and last in/first out. They test their new algorithm on several problems presented by Warszawski and Kuehn and Hamburger (1963), as well as a new group of larger problems with four items and as many as twenty warehouse locations. A heuristic approach for solving multi-item facility location problems is provided by Aggarwal et al. (1995) who address a composite problem consisting of maximum flow and minimum cost components. In addition, Pirkul and Jayaraman (1998) use heuristic procedures for solving a multi-item, multi-plant CFLP. More recently, Canel et al. (2001) formulate and analyze a more complicated multi-item, multi-period capacitated facility location model. This work presents the composite model and then suggests a three-phase solution algorithm which 1) identifies dominant facilities to be closed or opened for the duration of the analysis 2) uses branch and bound to solve the static multi-commodity phase and 3) uses dynamic programming to create a multi-period solution from solutions generated in phase two. The model is then applied to a small example problem with only two items, two factories, four potential facility sites, five customers, and three time periods. This model provides excellent example of the type of composite facility location models that are being assembled from simpler models already developed in the literature. It also emphasizes

the limitation on the problem size when trying to find an optimal solution for a composite problem with a great number of features and requirements.

Multi-Objective Facility Location Problems

In the literature on location analysis and resource allocation, problems and models that try to simultaneously satisfy more than one objective appear with much less frequency. However, it is not uncommon for logistics and operations professionals to encounter conflicting objectives in real world problem scenarios. For instance, limiting demand response times to customers may necessitate a large number of supply locations near customer markets; however, the need to minimize costs implies centralization of supply stocks and limiting the number of supply or distribution facilities. Methods for analyzing such multiple objective problems are described by Steuer (1986), Vincke (1992) and others. Some of the first location problems to use multiple objectives are formulated by Lowe (1978), Ross and Soland (1980), and Tansel et al. (1982). Additionally, Berman (1985) formulates a multiple objective approach for minimizing travel and lost customer costs. Like many multi-objective approaches this research uses weights to define the importance and tradeoffs between objectives. In a review of multiple objective location literature, Current, Min and Schilling (1990) state that minimization of cost and maximization of coverage are the two most common objectives used in a multi-objective approach. In addition, they state that most models use previous single objective models that are uncapacitated, static and with a single planning period and point towards the need for more research using stochastic inputs, capacitated storage

and flows and dynamic models with multiple periods. Other multiple objective work by Badri et al. (1998) uses integer goal programming to determine the number and location of fire stations and the demand areas they will serve in the City of Dubai. The model takes into consideration eleven strategic objectives and the optimum location of the model depends on which objectives are selected and the importance the decision maker gives to each objectives. This method is capable of generating a large number of flexible solutions and leaves any final decision making up to the practitioners utilizing the results. Additionally, Ogryczak (1999) recommends a distribution approach to multiple objective location problems which takes into account the entire distribution of customer service distances or times instead of merely analyzing the average distance (p-median approach) or the maximum distance (p-center approach). Kolli and Evans (1999) also apply multi-objective integer programming to solve location problems as they occur when trying to satisfy the conflicting objectives of franchisers and franchisees in the fast food industry.

Nozcik (2001) formulates a multi-objective facility location problem that is a combination of the fixed charge facility location model and the maximum set covering problem. This is accomplished by formulating the facility location problem with coverage constraints in order to satisfy both objectives. Solutions are accomplished using two Lagrangian techniques: allocation relaxation and decoupling relaxation. Two test problems (62 nodes and 102 nodes) are solved using these methods and the decoupling heuristic is found to be better in terms of speed and quality of the solution. In additional work, Nozcik and Turnquist (2001) apply similar methods to formulate a combined facility location and coverage model that includes a weight term in order to manipulate or

vary the tradeoff between minimizing uncovered demand and minimizing cost. The model is applied to an automotive industry problem in order to locate distribution centers to serve 698 demand locations in the United States. Solving the problem in several instances allows the authors to map an efficient frontier to graphically visualize the tradeoff between coverage and cost minimization. For all solutions the number of distribution centers is determined endogenously by the model; however, the coverage distance and weight term must both be determined exogenously and input into the problem by the modeler. The research by Nozcik and Turnquist provides an example of how multiple objective location decisions for storing and distributing inventory can be made using composite facility location models. This research is aimed at continuing this effort while additionally considering multiple items and inventory allocation decisions in a similar model.

Multi-Source Facility Location Problems.

The Facility Location Problem can also be differentiated by whether or not a demand point may be supplied or serviced by more than one location. In the single source version of the problem each demand can be supplied by one location and in the less frequent multiple source problem each demand by be supplied by more than one location. In additional work on developing heuristic methods for the p-median problem, Cooper (1967) is the first to recognize and model a multi-source approach where the demand at a single destination may be supplied from one or more source locations. This approach to location modeling is rarely seen in the literature. Work by Akinc and

Khumawala (1977) use branch and bound techniques to provide solutions to a capacitated warehouse location problem that allows multiple sourcing. In addition, research by Geoffrion and McBride (1978) and Nauss (1978) during this same time period allow for multi-source location problems, but very little research in the last twenty years makes an effort to address multi-sourcing as a major part of location modeling.

Location Covering Problems and the P-Center Problem

Other variations of the p-median problem include different objective functions that do not attempt to find the minimum sum or median location for the system. Instead, decision makers may be more concerned with minimizing the response time or distance to all of the destinations being serviced. Such an objective may be necessary for location emergency facilities or for industries with high stock-out costs. For example the, P-Center or Minimax problem attempts to minimize the maximum distance of any demand point from a supply location (Hakimi, 1964; Morrill and Symons, 1977) and thereby finds the geographic center for the locations being selected. In addition, covering problems have been developed that use a maximum distance constraint between the supply and demand locations (Toregas et al., 1972; and Church and ReVelle, 1974). The two most famous covering problems are the Location Set Covering Problem (LSCP) and the Maximum Location Covering Problem (MLCP). The LSCP attempts to minimize the number of facilities to cover a given set of demands and the MLCP aims to maximize the quantity of demand covered with a limited number of supply locations.

Unfortunately the two covering problems have their drawbacks and are not robust enough for a wide range of uses. The LSCP must have an exogenously determined distance constraint input into the model by the user, and the resulting number of supply locations determined by the model may be prohibitively large (Daskin, 1995). The MLCP relaxes the assumption that all demand locations be covered and assumes it is feasible to not service a certain percentage of the demand. This may not be realistic for emergency oriented problems such as the munitions prepositioning problem. Therefore, in order to not predetermine the number of facilities or the maximum coverage distance it may be necessary to use the P-Center problem which has occurred throughout the location literature starting with Hakimi (1964). The p-center attempts to find the geographic center solution for the problem. For the 1-center problem this solution attempts to find the center of a circle with the minimum radius that encloses all of the demand locations. Such a solution seeks to find locations that guarantee a certain level of service and does not guarantee a minimum cost solution as sought in the p-median problem. Objectives for the p-center problem are typically formulated as minimax problems where the optimum solution minimizes the maximum distance to the demand locations. Such a problem has been shown to be NP-hard by Kariv and Hakimi (1979) and is addressed in the literature by Minieka (1977), Handler and Rozman (1985) and many others. Most recently Ogryczak (1997) considers a lexicographic minimax approach to location problems including the p-center problem.

Resource Allocation Problems

The resource allocation problem is concerned primarily with allocating a scarce set of resources among competing activities. This includes the placement of emergency services resources and equipment and the stocking of strategic inventories at supply warehouses. These problems are usually modeled as either maximal covering problems or as minimax problems that attempt to minimize the maximum time in responding to any of the demand locations. Kolesar and Walker (1974) create such a model for relocating fire stations. Also, Kaplan (1974) studies military mission effectiveness and optimal ship loads based on the best allocation of resources. His research uses a maxi-min objective function with resources such as ammunition, food, manpower and fuel, and obtains solutions through simplified linear programming techniques. Brown (1979) studies resource allocation as it applies to the distribution of scarce water supplies and Agnihotri (1982) studies the distribution of a critical product among competing demands. Other research on resource allocation has considered multi-period problems (Luss and Smith, 1988) and resource substitution (Klein, et al., 1993). A comprehensive review of resource allocation literature is given by Katoh and Ibaraki (1998). More recently, Luss (1999) has developed a multi-item approach for resource allocation problems using a lexicographic minimax objective function that ranks the importance of a group of items in a CFLP and solves the problem iteratively starting with the most important item and working towards the least important item. Additional resource allocation work by Carvalho and Powell (2000) develops a multiplier adjustment method for resource allocation problems.

Research on resource allocation has also considered inventory allocation for many years. The roots of this problem can be traced back to the research by Clark and Scarf (1960) on distribution system design. Other work by Schwarz (1981) considers distribution system design and emphasizes that distribution systems must consider inventory cost structures. This work also does much to argue for the justification of the centralization of inventory stocks in order to minimize system wide costs. Similar research by Jackson (1988) extends the work of Swartz by analyzing the effect of pooling inventory in a central warehouse and restocking retailers during each period. This research develops a mathematical function for approximating the holding and shortage costs of centralized inventory policies and conducts a number of simulations to measure these costs for different inventory stocking scenarios. Additionally, Erkip et al.(1990) addresses inventory resource allocation issues as they relate to determining the right amount of safety stock to hold at a warehouse with correlated demands. Also, Meller (1995) identifies a method for identifying the necessary increases in demand needed to offset the fixed cost of building additional distribution centers. More recent work, by Glasserman (1997) addresses critical safety stock levels and develops bounds for stocking levels in a multi-echelon system. In addition, Rappold and Muckstadt (2000) address a distribution allocation problem with short production and transportation lead times.

Since the work of Brown (1979) and Agnihotri et al. (1982), little research has addressed the allocation of strategic resources or inventory as needed in military or public sectors with the exception of repairable inventory models and theory. Instead, the

majority of the research on inventory allocation addresses stocking policies and inventory centralization in multi-echelon retail inventory systems typically seen in the business world. However, the research and literature containing inventory modeling and resource allocation methods contains many aspects of facility location analysis and thereby lays the groundwork for developing combined or composite models that simultaneously address resource allocation and facility location as a single problem.

The Combined Location and Resource Allocation Problem

A limited amount of research has been conducted on problems that combine location analysis with resource allocation (Geoffrion (1979)). Benjaafar and Gupta (1998) conduct an analysis of production facility selection that focuses on the number of facilities to build and the subsequent capacity and product mixes to manufacture at each location. Additionally, Erlebacher and Meller (2000) formulate a Location-Inventory Problem that combines inventory ordering and carrying costs with an uncapacitated single-item facility location problem. Syam (1997) extends the capacitated facility location problem by restricting the number of locations that can be opened in a particular region. In addition, the model allows different capacity size options at the potential facility locations and therefore simultaneously considers the decisions of location and resource allocation. However, only three discrete capacity sizes are considered by the model limiting it significantly in comparison to a model which must consider a large number of different inventory stocking options. In addition, Antunes and Peeters (2001) provide heuristic solutions for the problem of locating school classrooms in Portugal by

solving a combined resource allocation and location problem. This work simultaneously considers the positioning of a resources and the selection of locations. However, the model does not consider multiple items or the logistics costs associated with resource positioning decisions. The work by Antunes and Peters, however, is a major step towards developing heuristics methods for solving such a combined model. More recently, Syam (2002) simultaneously addresses the problem of selecting location, shipment sizes, capacity and shipping cycle time in a capacitated facility location problem called the location-composition problem. This model includes inventory costs typically associated with the Economic Order Quantity (EOQ) model, includes high levels inventory turnover, and does not address strategic issues associated with coverage. However, the Syam model does allow for multiple sourcing of demand from more than one warehouse location and is the most recent step in the development of composite models that consider resource allocation as well as location decisions.

Modern Search Heuristics

Despite the fact that several advancements in exact algorithmic techniques have been accomplished during the last three decades, the use of heuristic methods continues to provide a source of solutions for complicated facility location problems. Geoffrion and Powers (1995) state that the majority of companies using location software models are using software programs with heuristic algorithms. Very few use software that implements exact algorithmic techniques because of the slow migration of exact techniques from the mainframe to desktop computers and the lack of Graphic User

Interfaces. Even though Geoffrion expects that the non-optimizing techniques will eventually fade from use, he does believe they will still be used to capture savings in computational time for larger problems and that heuristics techniques themselves will continue to advance in sophistication. These expectations have proven themselves during the past decade with the development of modern search heuristic techniques such as Tabu Search, Genetic Algorithms, Neural Networks, Ant Colony Search, and Simulated Annealing. Each of these techniques is widely documented in the literature and provides improvements in global optimization, pattern recognition and search routines for a variety of research fields and problem applications.

Tabu Search is attributed to the work of Glover (1977,1990) and is successfully applied to location and resource allocation problems by Brimberg and Mladenovic (1996), and others. In addition, Brimberg et al. (2000) compare heuristic methods for solving location problems using several heuristics including genetic algorithms and Tabu search. Genetic algorithms are equally well represented in the literature and have been applied to location problems by Houck and Jones (1996), Preston and Kozan (2001), and Jaramillo, Bhadury, and Batta. (2002). Neural Networks are less prevalent in the location literature but have been applied to resource allocation (Coit & Smith, 1996). Ant Search is a more recent heuristic search method attributed in large part to the work of Dorigo and Gambardella (1996) but has not yet been applied to location or resource allocation problems.

This research uses the final modern search heuristic listed above, Simulated Annealing. This method first used by Kirkpatrick, Gelatt, and Vecchi (1983) applies a

process first proposed by Metropolis, A. Rosenbluth, M. Rosenbluth, A. Teller and M. Teller (1953) to simulate the process of the annealing of metals to solve combinatorial optimization problems. Since then, this technique has been applied to location and resource allocation problems by Liu and Kao (1994), Ohlemuller (1997), McMullen and Strong (1999), and Antunes and Peters (2001). The process of simulated annealing is analogous to the metallurgical process of annealing in which the cooling of molten metal is controlled in order to insure the metal reaches an optimal crystalline structure. This state is desired in order to achieve correct physical characteristics such as the level of hardness or malleability in a metal. Similarly, the heuristic search method of simulated annealing uses a cooling schedule to control the decreasing probability of accepting an inferior solution in the simulated annealing search process. Periodically accepting an inferior solution allows the search to explore a greater portion of the feasible search area and helps to avoid being trapped at local minima. This ability helps the search obtain solutions that are much closer to the globally optimum solution similar to the actual process of annealing. The parameters for controlling the simulated annealing process and the implementation of this process in this research are presented in the Methodology. Descriptions of simulated annealing are presented in the literature by Kirkpatrick et al. (1983), and Eglese (1990). In addition, texts describing Simulated Annealing and combinatorial optimization in further detail are provided by Azencott (1992) and Aarts and Leenstra (1997).

Methodology

Air Force Problem Approach.

The components, procedures and responsibilities for the US Air Force's munitions pre-positioning program are outlined in Air Force Instruction 21-201, Chapter 15: The Global Asset Pre-Positioning (GAP) program. This regulation provides guidelines for the positioning and management of stocks of WRM munitions which are meant to be available to be transported to wartime locations in time of a conflict. More importantly, the Air Force Instruction designates that "Theater pre-positioned assets, although an integral part of GAP, are managed by owning theater commanders." This directive gives commanders in Europe, Southwest Asia and the Pacific the autonomy to position their WRM munitions where they believe they will provide the highest level of preparedness and were they can be properly stored and maintained (AFI, 2000). In addition, Air Force Instruction 21-201 does not provide a mathematical model for making pre-positioning decisions at either the global or theater level. However, such models are used by logistics planners in the US Air Force to analyze the munitions pre-positioning problem and related logistics decisions (Yost 2001, Synergy 2001). These combinatorial problems are addressed using mixed integer programming and goal programming in order to evaluate munitions positioning alternatives for the Air Force. However, no known model uses the heuristic methods used in this research. Such studies do provide an overview of the history and regulations behind the US Air Force positioning programs

and procedures (Johnstone 2002) and provide a baseline for the size of expected conflicts and the resulting munitions demand requirements for such conflicts.

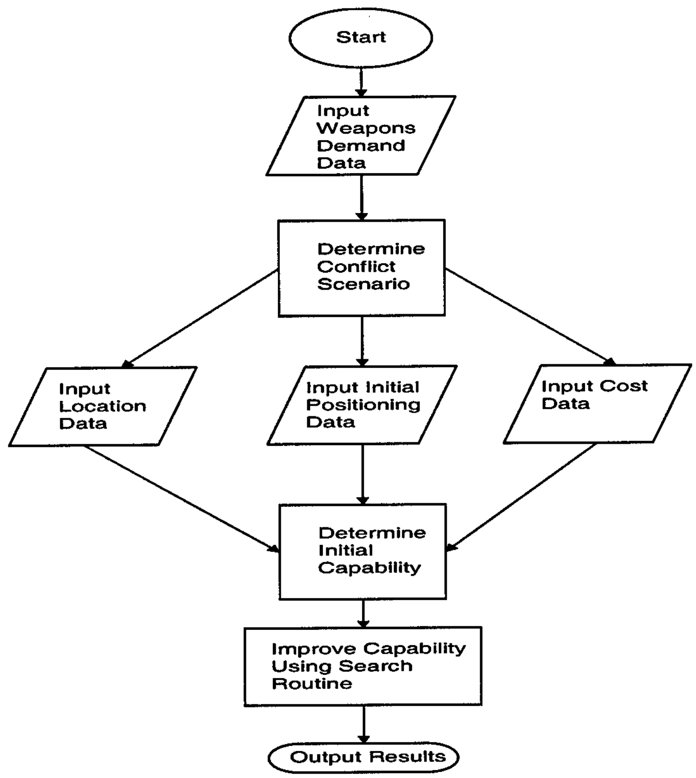

Figure 1. Research Process

Description of Data and Research Process.

Several types of data are acquired from US Air Force sources in order to initialize and run the model for this study. Location coordinates for possible supply and demand sites, munitions demand data, facility and transportation cost data, and a set of notional munitions inventory quantities are all used to develop a realistic model of the Munitions Pre-Positioning Problem. This data is used in the pre-positioning process of

this research described in Figure 1. Initially, a forecast of demand is determined for the munitions types under consideration for a predefined time period. Then, a particular conflict scenario (MTW, SSC or both) is selected and location data, cost data and munitions inventory data are input into the process. This combined information is compared to the preexisting positioning state to determine the initial ability of the Air Force to meet future demands as described by the model's objectives. In the final step, incremental changes are made using a search algorithm in order to explore different positioning options and to make improvements to the objectives of the model. The data sets necessary for this process are described in detail below and are listed in Appendix B.

Munitions Facility Locations and Costs.

Several references are used to identify military airbases and other known logistics locations that transport and store munitions around the world (Johnstone 2002, Recknor & Osborne 1998, Underwood and Bell 1999, US Air Force 2002). Initially, over one hundred potential munitions storage sites around the world were identified for possible inclusion in the study. These locations are a combination of current storage locations, past storage locations, and locations where the US Air Force has engaged in major conflicts or exercise during the last seven years. Next, the locations were cross-referenced with the US Air Force's Forceview Database at the US Air Force Wargaming Institute at Maxwell Air Force Base, Alabama, in order to verify the existence of military facilities and to acquire accurate longitude and latitude coordinates needed in the distance

computations of the model (Bush, 2000). This validation was necessary in order to verify the existence and exact location of smaller sites identified by the military installation's name instead of by a more common city name that might be found in a typical geographic information system. For running the model, the location set was reduced to exactly 100 potential locations dispersed throughout the three major overseas theaters of the Pacific Command, Central Command and European Command. No locations in the Western Hemisphere are included in this model, as the model does not address homeland defense, or possible military conflicts in Latin America. The only exceptions to this rule are the existence of munitions locations in Eielson and Elmendorf Air Force Bases in Alaska due to their importance for supporting possible operations in Northeast Asia. A list of the locations is contained in Appendix B.

Possible storage capacities for munitions storage facilities and the costs of constructing such facilities were obtained with help from the Air Force Logistics Management Agency, Maxwell Air Force Base, Alabama (McMillon, 2002). This organization provided military budgeting documents establishing the cost for a new Munitions Warehouse at approximately $1,000,000 dollars (Construction, 2001). Existing Air Force munitions storage areas in overseas locations might contain anywhere from 50 to 100 warehouses and each location also contains three to four munitions maintenance facilities to ensure the serviceability of the munitions when needed for war. Some existing sites such as Ramstein, Germany; Anderson, Guam; and Kadena, Japan have 100 munitions warehouses or more. However, in the model construction of new sites is limited to three different sized storage areas with up to 40 warehouses. Locations

in countries considered major allies and locations relatively close to existing US military locations are able to construct 40 warehouses. More distant locations are only allowed to construct 20 warehouses and the most remote locations are limited to 10 warehouses. Based on these numbers, the costs to open the munitions storage areas for the model is computed by multiplying $1.0 million by the number of warehouses needed and then adding an additional $5.0 million for munitions maintenance facilities and other necessary equipment and vehicles needed to run the site. Therefore, the maximum initial costs for munitions areas are $15.0 million for a small site, $25.0 million for the medium size site and $45.0 million for the largest site. Closing costs for the three storage area sizes are computed as 5% of the total opening costs for a site of the same size. These closing costs are added to the model since it is likely the US will not be able to sell or receive other compensation for the storage facilities at closed sites and will still have to pay for the redistribution of munitions and other equipment from the site.

The capacity for the three sizes of munitions areas is determined by taking into consideration the number of storage facilities and the tonnage of munitions stored at the location. The number of weapons and the total weight of munitions stored in an individual munitions storage warehouse depend greatly on the type of munitions being stored; however, approximately 10,000 tons of munitions of the types used in this study can be stored in a munitions storage area containing 40 warehouses (250 tons per warehouse). Therefore, capacity constraints in the model are calculated and verified by converting the number of each weapon type stored at a location into a total ton measurement and ensuring it does not exceed the limits set for each size of storage area.

The three capacities are linearly set at 2,500, 5,000, and 10,000 tons. The ton conversion rates for the weapons considered in the study are listed in Appendix B.

Combat Location Data.

Previous research by Johnstone (2002) contains notional munitions demand data for a Major Theater War in Southwest Asia and Small Contingencies in Europe and Northeast Asia developed with the help of US Central Command, Tampa, Florida. This study contains demands by individual weapon type and is therefore close in scope to this study. Other similar models measure all munitions by tons and do not consider individual munitions items (Abell et al., 2000). Therefore, the Johnstone data is used as a baseline for developing munitions demands for this study. However, several improvements and expansions of the data are made in order to provide a broader scope of munitions demand. First, two additional weapon types are added to the eight originally considered by Johnstone. In addition, a Major Theater War Scenario for Northeast Asia is created by doubling the size of munitions demands from the Southwest Asia MTW and assigning them to five current locations located in the Western Pacific Region. It is logical to make the new MTW twice as large since munitions demand would be much larger in this region to account for larger militaries and industries of potential enemies in this region compared to those in Southwest Asia. In addition, a Small Scale Contingency is developed for the Southwest Asia region in the same manner by using a similarly sized demand for munitions from another SSC and assigning it to three commonly used munitions storage areas in Middle East. Using the MTW and SSC

scenarios, three different demand sets are built containing different mixes of possible conflicts. The first scenario called "MTW" contains six MTW conflicts, two SSC conflicts and a group of randomly generated demands (20% of the total demand) representing unexpected conflicts during the planning period. The second scenario named "Mix" consists of four MTW, four SSC, and the group of randomly generated demands. Finally, the third scenario, "SSC" consists of six SSC, two MTW and the randomly generated locations. These three different problem scenarios are also varied by number of total demand locations (n=50 and n=100). This results in a total of six different problems for the study. The Design of Experiment section fully describes the methodology and related experiment for these problems and each demand set is listed in Appendix B.

The initial locations and munitions positions are based on munitions numbers and locations used in the Air Force's Global Engagement war-game conducted at Maxwell AFB, in November, 1999 (Air Force Logistics Management Agency, 1999). Twenty sites from the three major overseas regions (Pacific, Europe, and the Middle East) are selected as the initial supply locations for the model, and the supply quantities for each of the ten weapons for the study are equal to the munitions quantities used by the war-game for the three major regions.

Munitions Transportation Costs.

Actual data for shipping tons of munitions in the different theaters of the world was provided to this study by Headquarters US Air Force Material Command's

Requirements Division for Munitions (Panzer, 2002). This data establishes the relative transportation costs for the three overseas theaters and shows the average surface transportation rates for the European Theater, Central Theater, and Pacific Theater for Fiscal 2000. The air cargo rates for defense transportation were acquired from Headquarters US Air Force Air Mobility Command (AMC), Scott AFB, Illinois (US Air Force Air Mobility Command, 2002). As expected, these rates are significantly higher than the surface transportation rates. Using these sources, the cost data for air and surface transportation for this study were established and can be seen in Table 1. Overall, these costs more accurately reflect the costs for different modes of transportation used by the Air Force transportation system around the world and highlight the difference in costs between regions and the expense of air transportation.

Table 1

Transportation Costs ($/ton/mile)

	Command			Distances			
	Euro	Cent	Pac	<500	500-1000	1000-4000	>4000
Surface Rate	.12	.18	.30	-	-	-	-
Air Rate	-	-	-	4.00	2.00	1.00	.75

Finally, the location and region for determining transportation rates is based on the shipping origin of the munitions. For example, costs for munitions shipped from Hickam AFB, Hawaii to Ramstein AFB, Germany, are calculated using the Pacific Region's transportation rate for munitions tons if the move is my surface transportation. Air transportation rates are simply calculated based on the distance between the two locations.

Formulation of Combined Inventory Allocation and Facility Location Problem.

The variables for describing the combined inventory allocation and facility location problem are listed in Table 2. This problem is considered static and does not consider the changes from one time period to the next as seen in a dynamic model. Even though the conflicts may be thought of as occurring in different time periods or years, they are aggregated into one common planning period (m=50 or m=100) and no one conflict is said to occur before another during this planning period. Also, the problem is said to be zero-echelon, meaning that production plants are not considered and that only the location of distribution facilities is considered. In addition, no hierarchy of these distribution locations is used or implied in the model. Since the problem is multi-objective, two different objective functions are presented where the first objective is a multiple item, capacitated facility location problem similar to the original single-item formulation of Akinc and Khumawala (1977).

Table 2

Problem Variables

Abbreviation/Symbol	Definition
n	Number of potential supply facility locations
m	Number of combat demand locations
q	Number of different weapon types
p	Number of distinct conflicts
i	Index for the set of potential supply locations n
j	Index for the set of combat demand locations m
k	Index for the set of inventory weapon types q
A_{ij}	Total distribution cost of shipping all required munitions from location i to combat location j
C_{ijkt}	Per unit distribution cost of shipping item k from supply location i to combat location j by mode of transportation t, where $t = 1$ represents air transportation and $t = 2$ represents surface transportation
X_{ijk}	Number of units of item k shipped from location i to combat location j
O_i	Fixed cost of opening location i
CL_i	Fixed cost of closing location i
Y_i	1 if supply location i is opened from a closed state, and 0 otherwise
β	Fractional number of units allowed to shipped by air, set at .10
Z_i	1 if supply location i is closed from an initial open state, and 0 otherwise
S_{ik}	Number of inventory units of supply of item k at location i
D_{jk}	Total demand at combat demand location j for item k
U_{jl}	1 if combat demand j occurs during conflict l, and 0 otherwise
T_i	Total physical capacity of supply location i in tons
W_k	Number of tons per unit of weapon type k
$Dist_{ij}$	Distance between supply location i and demand location j
$Circ_i$	Circuitry routing factor for surface transportation from location i
r	Radius of the earth, equal to approximately 3956 miles
t	Mode of transportation, 1 if air and 2 if surface transportation
φ	Latitude of a location
γ	Longitude of a location

This problem is a mixed integer program with *n* supply facility locations, *m* number of combat demand locations, *q* number of different weapon types, and *p* number of distinct conflicts. The formulation is as follows:

Minimize $$\sum_i \sum_j A_{ij} + \sum_i (O_i Y_i + CL_i Z_i) \tag{1}$$

With

$$A_{ij} = \sum_k (C_{ijk1} * (\beta * X_{ijk}) * Dist_{ij}) + \sum_k (C_{ijk2} * ((1-\beta) * X_{ijk}) * Circ_i * Dist_{ij}) \tag{2}$$

$i = 1$ to n, and $j = 1$ to m

$$Dist_{ij} = r * (2 \tan^{-1}(\sqrt{b}, \sqrt{1-b})) \tag{3}$$

$$b = \left[\sin\left(\frac{(\varphi_j - \varphi_i)}{2}\right)\right]^2 + \cos(\varphi_i) * \cos(\varphi_j) * \left[\sin\left(\frac{(\gamma_j - \gamma_i)}{2}\right)\right]^2 \tag{4}$$

Subject to

$$\sum_i X_{ijk} = D_{jk} \qquad j = 1 \text{ to } m,\ k = 1 \text{ to } q \tag{5}$$

$$X_{ijk} \le S_{ik} \qquad i = 1 \text{ to } n,\ j = 1 \text{ to } m,\ k = 1 \text{ to } q \tag{6}$$

$$\sum_i D_{jk} U_{jl} \le \sum_i S_{ik} \qquad k = 1 \text{ to } q,\ 1 = 1 \text{ to } p \tag{7}$$

$$T_i \ge \sum_k S_{ik} W_k \qquad i = 1 \text{ to } n \tag{8}$$

$$X_{ijk} \ge 0 \qquad i = 1 \text{ to } n, j = 1 \text{ to } m \tag{9}$$

$$Y_i \in 0 \text{ or } 1 \qquad i = 1 \text{ to } n \tag{10}$$

In the objective function (1), the first term shows the intent to minimize the total distribution costs A_{ijk} for shipping the required munitions from their supply locations *i* to the location for each demand occurrence *j*. The A_{ijk} values are computed using (2) where β is the fraction of the total requirement for each weapon type *k* that uses air transportation costs, C_{ijk1}, and the remaining (1- β) uses the surface transportation cost rate, C_{ijk2}. Additionally, the circuitry routing factor $Circ_i$ is used to adjust distances for the mode and quality of surface transportation available at supply location *i*. All distances are calculated with the Haversine distance computation in (3) and (4) which uses the longitude and latitudes of the two locations, the radius of the earth *r* and several trigonometric calculations to compute the distance between two locations on the earth's surface (Sinnott, 1984). The second term adds facility opening and closing costs to the objective function. The first constraint (5) requires that all of the demand for a particular weapon type *k* at each demand location *j* is filled by shipments from the supply locations. In addition, this constraint allows for multiple sourcing of munitions from more than one supply location *i*. Next, constraint (6) insures that the number of units of weapon *k* shipped from location *i* does not exceed the total inventory of *k* currently positioned at this location. Constraint (7) insures that the demand for each weapon *k* at the demand locations *j* belonging to conflict *l* does not exceed the total quantity of weapons stored at all locations *i*. This constraint insures no shortfalls in the solution of the problem. The next constraint (8) verifies that the quantity of all weapons types *k* measured in tons and stored a location *i* does not exceed the total ton capacity T_i for the supply location.

Constraints (9) forces non-negativity of the quantities shipped and stored by the model, and constraint (10) insures that variables Y_i and Z_i are constrained to the binary integer values 0 or 1.

Conflicts in the different problem sets contain either three, four or five distinct combat locations. It is important to note that the location of a combat occurrence *j* may occur more than once in a planning period *m* as long as the location does not occur more than once in any conflict *l*. For example, in the MTW scenario, Incirlik Airbase is the second (*j*=2) combat demand location and is also the twelfth combat demand location (*j*=12). However, this does create a violation since *j*=2 is a part of the first conflict (*l*=1) and *j*=12 is apart of the third conflict (*l*=3). A complete list of the conflict demand locations and the conflict numbers they belong to are listed in the problem demand sets in Appendix B. In addition, the problems have been prescreened to insure no conflict demand location occurs more than once in a conflict to preclude additional constraints in the formulated model.

Formulation of the second objective is a variation of the p-center problem on a network and is formulated as the minimax objective,

$$\min \ \max \left\{ \frac{\sum_i \sum_k \beta X_{ijk} Dist_{ij} + \sum_i \sum_k (1-\beta) X_{ijk} Dist_{ij} Circ_i}{\sum_k D_{jk}} \right\} \quad \text{for } j = 1 \text{ to } m \qquad (11)$$

This objective minimizes the maximum average shipping distance for all of the munitions required by any location j. For example if $m = 50$, the average shipping distance for all of the weapons required by each of the fifty combat demand locations is calculated and then the maximum value is minimized by the objective function. This objective is aimed at trying to provide service equity to each of the demand locations m and in many instances works against the mini-sum (median) objective found in the first objective for the problem.

As detailed in the Design of Experiment section, the model finds solutions that satisfy both objectives from an initial starting position with a number of already opened supply locations and a set of predetermined initial inventory quantities. However, the model is not considered a conditional facility location model since none of these locations is required to be in the final solution. Instead, the model is allowed to close any or all of the initial facility locations and move inventory to newly opened supply locations as dictated by the solution algorithm. Additionally, the model endogenously decides which locations to open and close and may decide on any final combination of supply locations up to the total number of potential supply locations n for the problem.

Combinatorial Aspects of the Problem.

The number of feasible solutions to the problem and the time it takes to find a globally optimal solution are substantial for the problem studied in this research. Both the facility location problem and p-center problem are considered NP-Hard and no known polynomial time efficient algorithms are known to solve them (Kariv and Hakimi, 1979).

The number of combinations for facility selection alone is considerable. For example, in a problem with n= 50 possible supply locations, the number of combinations of 25 supply locations (r) is equal to,

$$C_{n,r} = \frac{n!}{r!(n-r)!} = \frac{50!}{(25!)(25!)} = 126{,}410{,}606{,}437{,}752 \qquad (12)$$

However, in the problem considered in this study we do not set a fixed value for r, therefore the total number of combinations just for facilities is the equal to the summation of all of the values computed by equation (12) for r = 1 to n. In addition, at each of these unique facility location combinations the inventory quantities S_{ik} are varied by the algorithm. This adds a great number of additional combinations of feasible solutions to the problem. Computational ability prevents actually calculating the total number of combinations of combined inventory positions and facility locations. Regardless, it is understandable that heuristic methods are needed to find improved solutions to the problem. The program developed for generating solutions to the Munitions Pre-positioning Problem for this study is the heuristic algorithm Simulated Annealing.

Simulated Annealing Algorithm.

Simulated Annealing is based on the physical properties of the metallurgical practice of annealing. During this process a metal is heated and then slowly cooled in an attempt to find the crystal structure that provides physical properties desired in the metal.

Typically, annealing is used to increase hardness in a metal or to reduce brittleness. Analogous to this technique, Simulated Annealing searches the set of feasible solutions to a problem by occasionally accepting an inferior solution to the problem in order to eventually find an improved condition and subsequently prevent being trapped at a local minimum for the problem. The probability of accepting an inferior solution is controlled by a parameter referred to as the temperature. The initial temperature for the search is typically set at a relatively high value in order to provide a high probability of accepting an inferior solution at the beginning of the search. This allows the search to explore different regions of the search area and to avoid local optima trapping. However, as the search progresses the temperature is slowly decreased using a cooling rate and the probability of accepting an inferior solution is consequently decreased as well. This ensures that towards the end of the search, the model concentrates on typically accepting only actual improvements to the current objective function value. The search ends when the temperature reaches a predetermined final temperature value and the best solution is output by the model. The success of this technique for finding formidable solutions to combinatorial optimization problems has been well documented by researchers, and the technique has not yet been widely used by location analysts for solving complex location problems.

The simulated annealing algorithm and related C++ programming code used to generate solutions for the model is listed in Appendix A. Examination of this Appendix reveals that the performance of the Simulated Annealing search methodology is largely dependent on several variables. These variables include the cooling rate, initial and final

temperatures, the number of iterations at each cooling level, and the probability of accepting an inferior solution. For this study, the cooling rate is held constant at 95% a value commonly used in simulated annealing procedures (Azencott, 1992). In addition, the initial temperature, a control parameter, is set equal to 25, the final temperature is set to 1 and the number of iterations at each cooling level is 5,000 (McMullen and Frazer, 2000) a value found to be sufficient in pilot-testing. All runs of the model in this study use these values for accomplishing simulated annealing. Using these parameters the model accomplishes 5,000 iterations and then reduces the initial temperature of 25 by multiplying it by the cooling rate of .95 to obtain a new temperature of 23.75. Then additional cycles of 5,000 iterations are accomplished repetitively reducing the temperature each time until the final stopping temperature, $T_F = 1$ is reached. An additional simulated annealing parameter is the probability of accepting an inferior solution. This probability is initialized in the model by setting a Boltzman constant β such that the probability of accepting an inferior solution is set to a given percentage at the initial temperature. This relationship is defined using the variables in Table 3. The Boltzman constant is determined by the initial temperature and the desired probabilities and is formulated as:

$$\beta = \frac{-(E)}{\ln(PA_1)Temp_1} \tag{13}$$

The Boltzman constant is used in order to allow the user to maintain control over the search. For example, if the user wants to provide a 4% chance of accepting a solution,

which is 3% inferior to the current solution, at the initial temperature of 25, the Boltzman constant is determined to be .0003728 using (13). This fixed constant in the acceptance function implies that small increases in the objective function are more likely to be accepted than larger increases. In addition, as the model runs through successive iterations of the search, the temperature declines and the fixed Boltzman constant insures that the probability of accepting inferior solutions decreases. Towards the end of the search, the probability of accepting an inferior solution approaches zero and the algorithm concentrates on accepting only actual improvements to the objective function.

Table 3

Boltzman Variables

Abbreviation/Symbol	Definition
PA_1	Probability of acceptance at the initial temperature
(E)	Percentage difference between the current solution and the inferior solution being considered
T_1	Initial temperature
T_F	Final temperature

This research uses an acceptance probability of 4% for solutions up to 4% inferior at the initial temperature of $T_1 = 25$.

Generation of Solutions.

The program for generating solutions is outlined in Figure 2. This process begins by initializing the simulated annealing parameters for the problem and inputting the initial locations and inventory quantities for each weapon type at each location. This inventory positioning is then used to determine the initial values for the two objective functions (cost and coverage) prior to starting the simulated annealing process. After verifying the temperature, the first important decision is to determine whether the model will explore a facility location change or transfer inventory from one current location to another location within the capacity constraints of the supply locations. The probability of the iteration being a supply location change is set equal to 5% and all other iterations are inventory transfers. Variation of this value as an experimental value in further research is also warranted; however, for this research the value is fixed to not counteract the effects of the experimental factors. An inventory transfer is accomplished by first selecting at random one of the weapon types for a transfer (equal probability for each weapon type). Then two locations are selected for the transfer. The first location must have inventory of the weapon type to move and the second location must not already have reached its capacity for all weapons stored. The transfer is accomplished by simply subtracting the transfer quantity at the first location and adding the transfer quantity at the second location. The size of the transfer quantity is an experimental factor used in this research since it is believed that transfer sizes larger than one might provide a significant improvement in the cost and coverage values found by the search algorithm.

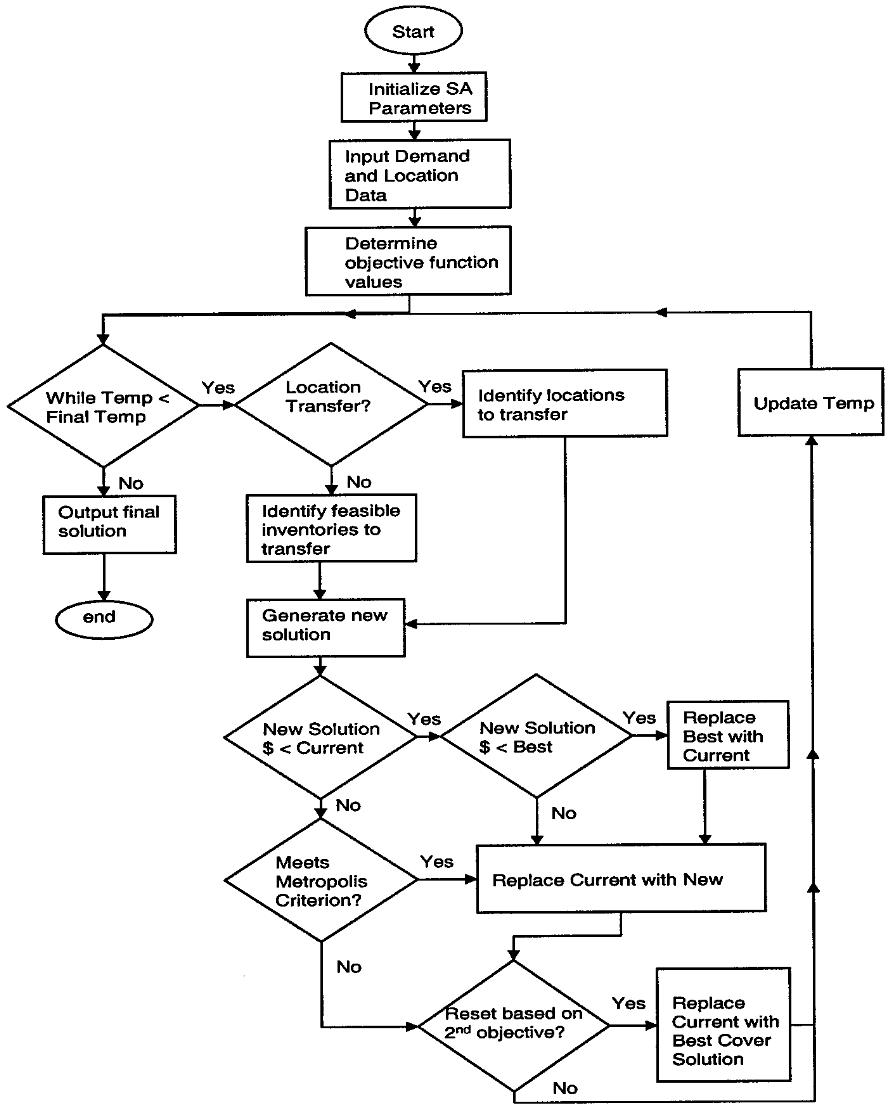

Figure 2. Flowchart of Simulated Annealing Model

The location transfer is a much more aggressive type of change than a simple inventory transfer. First, a potential location not already being used as a munitions storage location is selected at random as the site to open from all of the available potential sites. Then a current supply location is selected at random to be closed. The

location transfer is accomplished by moving the entire inventory for all weapon types at the current location to the new location. This change results in hundreds or even thousands of munitions being transferred at once and results in both closing and opening costs being added to the objective function (2). The aggressiveness of these location transfers is the primary reason for maintaining the probability of a location transfer at only 5%.

After accomplishing either an inventory transfer or location transfer, the model computes new values for both objectives. The new value for the first cost objective is used for comparison to previous solutions. If this value is less than the initial solution, then the new solution is kept as the current solution. If not, the metropolis criterion is explored using the previously described probability of accepting an inferior solution. This allows the possibility of accepting the new solution as the current solution even if it did not result in a cost improvement. The model also tracks the best cost solution found throughout the entire search and its corresponding coverage value.

During each iteration, the maximum average coverage value is also calculated. After a preset number of iterations, the search is reset to the solution with the best coverage value yet found. The search and generation of new solutions begins again from this point. In this manner, the simulated annealing search runs primarily on the first objective (minimum cost); however, the search is continuously redirected back to the search area where the minimum maximum coverage value has been discovered. If this reset occurs after only a small number of iterations, the search for solutions might be confined almost entirely to finding improved coverage values. However, if the reset is

done after an extremely large number of iterations, then it is believed that the search will be almost exclusively based on the first objective. Determining the number of iterations to compute prior to resetting the search to the best coverage value is an additional experimental factor for this research.

Example Problem.

The generation of solutions is easier to understand when analyzing a small example problem with three supply locations, five combat locations, and two weapon types. The five demand locations are each a part of a single conflict *p*. Initially, there are eight units of each weapon at location 1, four units of each weapon at location 2 and

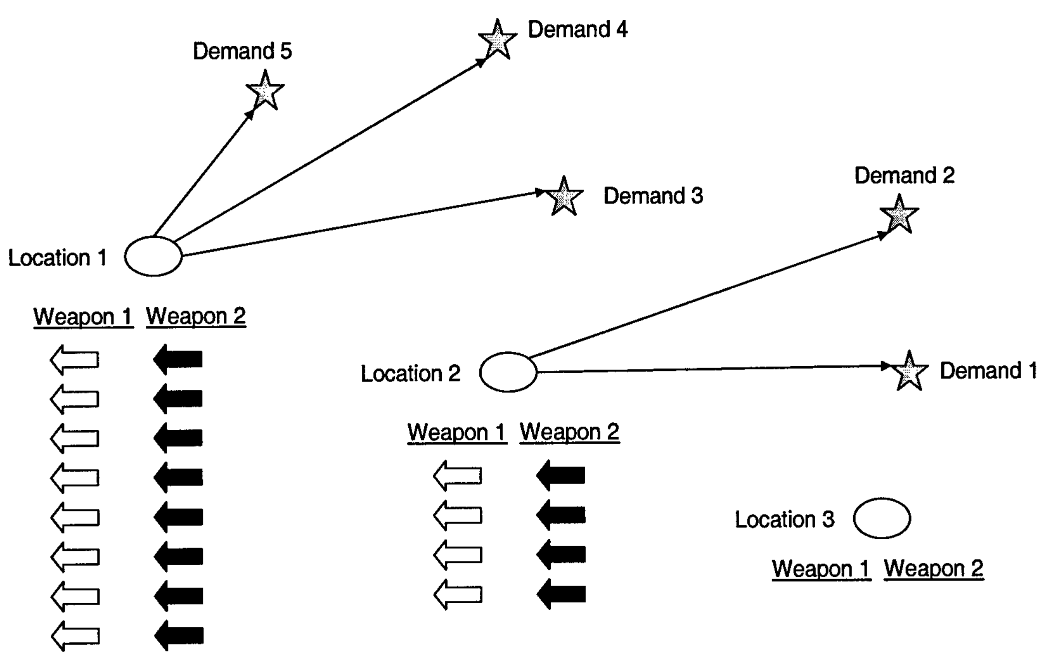

Figure 3. Example – Initial Position

location three is currently closed as seen in Figure 3. Also, the five combat locations each require two units of each weapon type and each supply location has a capacity of ten tons. Using this initial positioning information and the geographic coordinates for eight European locations, the initial objective value from (1) is \$1574.34. In addition, the solution results in a maximum average coverage value of 504.6 miles from (11). In the first iteration of the model, an inventory transfer is accomplished moving one unit of weapon 2 from location 1 to location 2 as seen in Figure 4. This transfer allows demand location 3 to receive one unit of weapon 2 from location 2 and one unit from location 1. This solution results in a cost of \$1201.20 and a maximum average coverage value of 320.1 miles. The next iteration, a location transfer

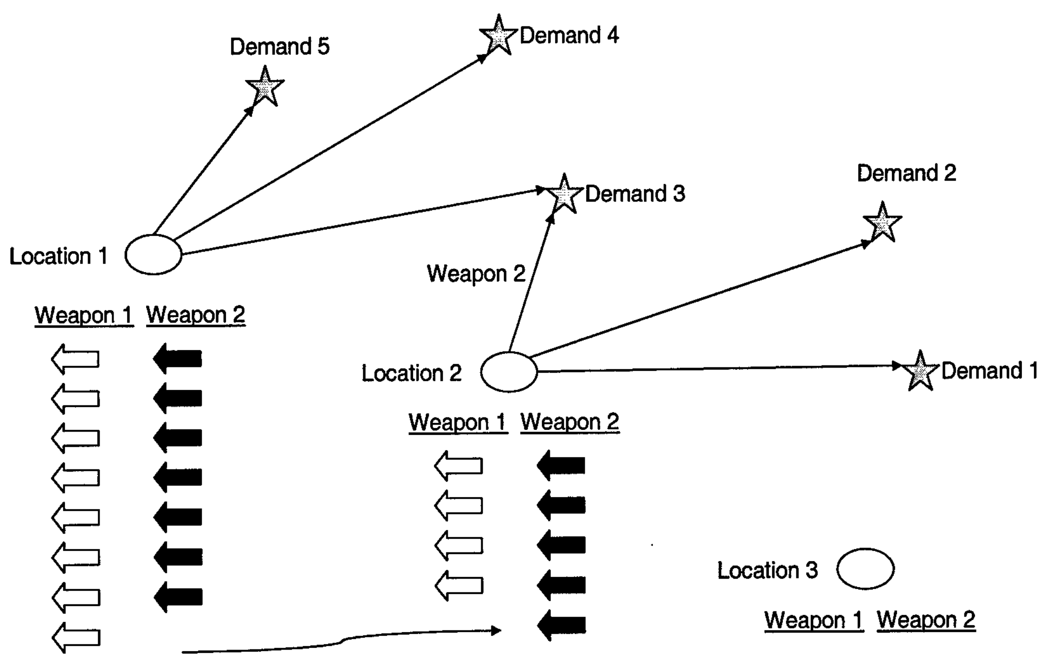

Figure 4. Example – Inventory Transfer

involves transferring all of the inventory currently at location 2 to location 3 as seen in Figure 5. This new positioning results in objective function values of $909.17 and 291.4 miles. Continuing in this manner, inventory and location transfers are accomplished as directed by the simulated annealing algorithm. Solutions that generate actual improvements in objective (1) are accepted and sometimes inferior solutions

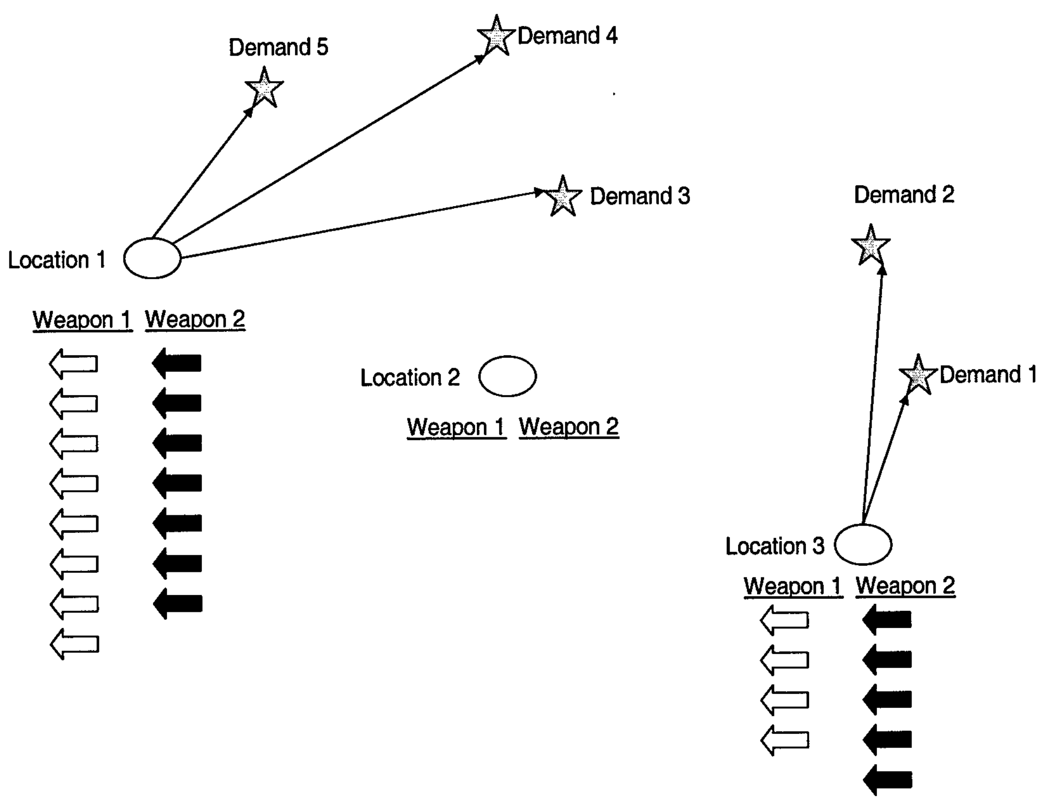

Figure 5. Example – Location Transfer

are accepted in order to keep from being trapped at a local optimum. Throughout the search, the second objective (11) is generated for each solution and periodically the search is redirected or reset back to the solution that has found the best value for (11) and

the search begins again. In this manner, a number of feasible solutions are generated and the best solutions (those with minimal cost in a particular coverage range) are captured and portrayed in an efficient frontier. The efficient frontier allows the decision maker to visualize the tradeoff between the two competing objectives of cost and coverage. An example with twenty solutions found using the model is seen in Figure 6. The initial solution in Figure 6 is inferior to all of the solutions in terms of coverage. However, increases in coverage result in the majority of the other solutions having an increased cost value. As depicted by the arrow, preferred solutions are those in the bottom left of the chart which result in a simultaneous decrease in the maximal average coverage value and the total cost.

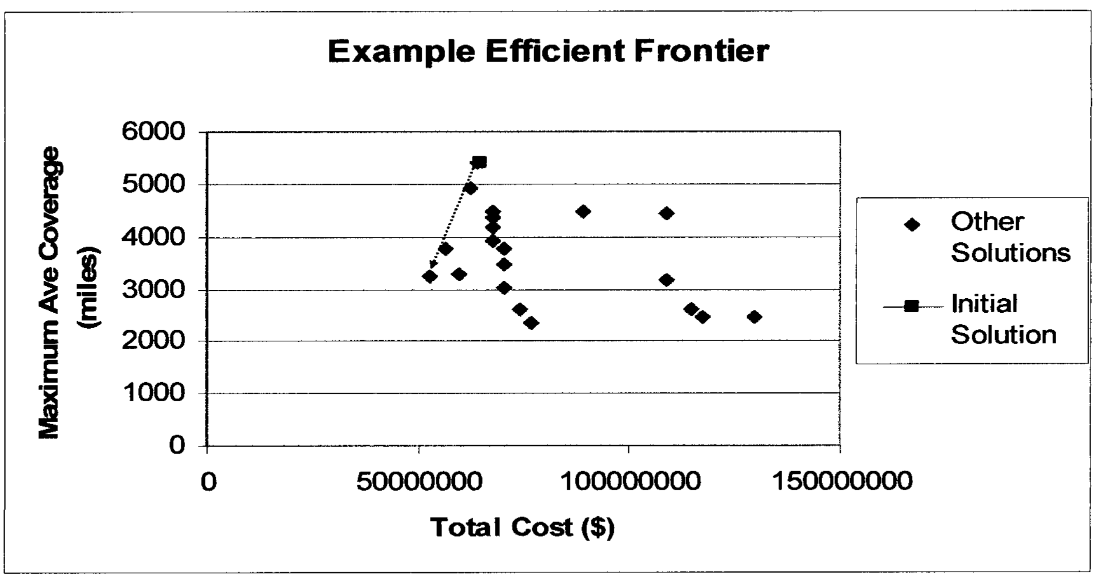

Figure 6. Example Efficient Frontier

Design of Experiment

An experiment is conducted in order to assess the model's ability to find solutions to six different problems that satisfy both objectives of the problem. This experiment is intended to answer a number of predetermined research questions. In addition, a structured design of the experiment is established in order provide evidence for answering the research questions in a manner where the results can be evaluated using statistical methods.

Research Questions.

The primary purpose of the research is to find solutions to the Munitions Pre-positioning Problem while insuring that the multiple objective nature of the problem is adequately addressed. In addition, it is the intent of the research to experimentally explore what techniques might improve the search algorithm's ability to find improved solutions to the problem. With these purposes in mind, several specific research questions are established.

1. Can the proposed Simulated Annealing solution algorithm find efficient solutions to the model of the Munitions Pre-positioning Problem that simultaneously improve both objectives of the model in comparison to the initial solution? This question is answered by first mapping an efficient frontier of the solutions in order to identify the trade-off between the two objectives. From the efficient frontier, a "best" solution is selected for further analysis by determining which of the identified solutions provides the best total

percentage reduction in both cost and coverage equally weighting the importance of the two objectives. These best solutions are then compared to the cost and coverage generated by the initial solutions.

2. Do the results for the dependent variables, cost and coverage, depend on the problem configuration? In other words, are there primary effects for the problem configuration research factor? Since the problems vary in the type and number of conflicts, it is thought that the dependent variables will also vary depending on the configuration of the problem. This research question is analyzed using multiple analysis of variance (MANOVA) and any main effect differences in the two dependent variables resulting from the problem configuration levels are analyzed using Tukey's pairwise confidence intervals.

3. Do the results for the dependent variables cost and coverage depend on the transfer size selected? The number of munitions moved during each inventory transfer may significantly impact the ability of the algorithm to find improved solutions. It is believed that the level of this factor will have a significant impact on the cost and coverage values for the solutions generated. The effects of the different levels of transfer size are analyzed using MANOVA and any main effect differences in the two dependent variables resulting from the transfer size levels are analyzed using Tukey's pairwise confidence intervals. Relevant treatment mean differences are examined using intervals developed using the Bonferoni procedure (Neter, 1996)

4. Do the results for the dependent variables, cost and coverage, depend on the reset frequency? How often the solution resets itself to the best coverage value is also believed to affect the cost and coverage values of the solutions. It is believed, that if the algorithm resets itself to the best coverage solution after only a few iterations then it may concentrate on finding best coverage solutions, whereas if the number is quite large the algorithm might be concentrate solely on improving the cost objective. The different levels of reset frequency are analyzed using MANOVA and any main effect differences in the two dependent variables resulting from the reset levels are analyzed using Tukey's pairwise confidence intervals. Relevant treatment mean differences are examined using intervals developed using the Bonferoni procedure.

Experimental Factors.

The experiment consists of six distinctly different problems in order to represent a broad range of possible demands on the model. There are three different conflict types (MTW, SSC, and Mix) intended to represent a range of different conflict demands. In addition, the size of the planning period is also a major consideration since the model is static and does not try to dynamically depict conflict occurrences in a temporal manner. Two different size problems are considered: one with 50 total demand locations (*m*) and another with 100. These two major differences result in six distinctly different problems for the factor "problem configuration". The differences in cost and coverage for the different problems as well as the models ability to find solutions for all six problems are addressed in the research questions. The model's ability to find improved solutions is

tested with two different factors. The multiple objective reset frequency is set at three different iteration levels and the transfer size is set at three different levels listed in Table 4. The combination of these two factors with the problem configuration factor results in a 6 x 3 x 3 full factorial design with 54 factor-level cells.

Table 4

Experimental Factors

Factors			
Problem #/Type - Locations		Transfer Size	Reset Frequency
1	MTW – 50	5	50
2	MTW – 100	25	150
3	SSC – 50	75	450
4	SSC – 100		
5	Combination – 50		
6	Combination – 100		

In order to gain a full understanding of the variation and standard deviations involved in the modeling process, 40 separate computer runs are generated for each cell. This results in 54 x 40 = 2160 runs of the model in order to generate the necessary data to complete the experimental design and establish statistical results. The conclusions for the study are then formed from these experimental results.

Results

Solutions for the Munitions Pre-Positioning problem are generated using a C++ computer program (Appendix B) running on an AMD Athlon 4 processor with 256 MB of RAM and operating at 900 MHz. The program recorded the minimum cost solution in eight individual coverage ranges for each of the forty runs for each experimental cell. The variety of solutions and the tradeoff between the two objectives can be seen in the efficient frontiers mapped for each problem. An efficient frontier of the solutions for Problem 1 is seen in Figure 7.

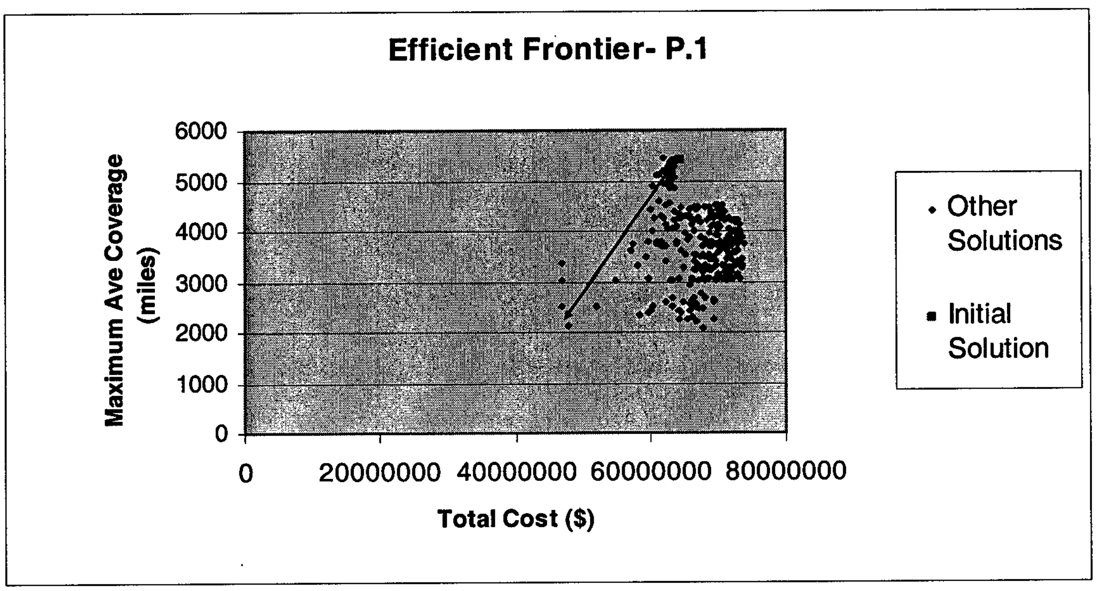

Figure 7. Efficient Frontier – Problem 1

The efficient frontiers developed for each of the remaining five problems are provided in Appendix C.

The solution from each run which was deemed to be the best solution in terms of overall improvement of the two objectives is used in the statistical analyses to answer the research questions for the study. The two objectives are equally weighted in determining the best solution from each run by simply averaging the percentage improvement for each objective. This equal weighting of objectives is used as a baseline of comparison and does not imply that the best method for comparing cost and coverage for munitions positioning should be weighted in this manner. Instead, individual decision makers must make their own decision regarding the proper weighting of cost and coverage objectives. The use of different weightings provides an additional opportunity for future research

Simultaneous Improvement of Cost and Coverage

The first research question in this dissertation addresses the ability of the program to find solutions that improve both cost and coverage in comparison to the initial solution for the problem. During the experimental runs, the program was able to find many individual solutions that simultaneously improve both objectives. In recording the success of the model to find improved solutions, the number of runs that found a solution that improved both objectives is recorded, as well as the average percentage improvement of the solution when the two objectives are weighted equally. In addition, the solution which provides the highest weighted improvement and simultaneously improves both objectives is considered the best solution for the experimental cell. It should again be

noted that the initial solution used in the study is based on a validated pre-positioning set used by the Air Force during a congressionally mandated war game conducted at Maxwell AFB, AL during November 1999. Therefore, improvements to the initial solution should not be considered trivial and should be considered a sincere estimate of improvement over current methods of positioning. The number of improved solutions and the best solutions for Problem 1 are provided in Table 5.

Table 5

Simultaneous Improvement of Objectives – Problem 1
Initial Solution (Cost-$64.59 million, Coverage- 5385 miles)

Factors				Best Solution Found		
Transfer Size	Reset Frequency	Number Improved	Ave % Improved	Coverage	Cost	Weighted % Improve
5	50	27	.399	5308	64.05	1.13
	150	26	.173	5356	64.16	.61
	450	24	.138	5369	64.22	.43
25	50	32	1.814	3621	64.59	16.39
	150	18	1.178	5176	64.11	2.69
	450	16	.683	5270	63.93	1.58
75	50	26	5.891	3604	57.33	22.17
	150	21	7.735	2510	46.75	40.51
	450	20	14.293	2115	47.82	43.35

For each of the nine experimental cells, approximately twenty to thirty of the forty runs found a solution that simultaneously improved the two objectives in comparison to the initial solution for Problem 1. However, the quality of the best solutions varied greatly.

For a transfer size of 5, the best solutions made only small improvements of approximately one percent or less. For a transfer size of 25 and a reset frequency of 50, the best solution resulted in an improvement of approximately 16.39%; however, the bulk of this improvement resulted from improvement in only the coverage objective. Solutions with a transfer size of 75 resulted in greatly improved solutions in comparison to the initial solution, and improvements were made to both objectives. A transfer size of 75 and a reset frequency of 450 found the best solution for Problem 1 with a weighted improvement of 43.35%.

The solutions for Problem 2 are listed in Table 6. Similar to the first problem, Problem 2 is based on the occurrence of Major Theater Wars (MTW); however,

Table 6

Simultaneous Improvement of Objectives – Problem 2
Initial Solution (Cost-$128.43 million, Coverage- 5385 miles)

Factors				Best Solution Found		
Transfer Size	Reset Frequency	Number Improved	Ave % Improved	Coverage	Cost	Weighted % Improve
5	50	10	.100	5379	128.07	.19
	150	10	.085	5373	128.33	.15
	450	10	.061	5379	128.37	.08
25	50	9	.432	5299	127.98	.97
	150	8	6.69	2597	126.69	26.57
	450	14	19.46	2664	121.77	27.85
75	50	16	6.09	2400	123.80	29.52
	150	27	24.29	2286	108.18	36.66
	450	39	31.80	2527	99.75	37.70

it includes 100 combat locations. The results indicate the difficulty of obtaining simultaneous improvement to the objectives for this larger problem. With a transfer size of 5, only ten of the forty runs find a solution that improves both objectives and the improvement is almost negligible. A transfer size of 25 does slightly better and with a reset frequency of 150 and 450, solutions are found that improve the coverage objective by as much as 50%. However, the number of improved solutions and the size of improvement increase greatly when the transfer size is set to 75. The best solution for Problem 2 is found with a transfer size of 75 and a reset frequency of 450, and it results in a weighted improvement over the initial solution of 36.66%.

Table 7

Simultaneous Improvement of Objectives – Problem 3
Initial Solution (Cost-$28.27 million, Coverage- 5385 miles)

Factors				Best Solution Found		
Transfer Size	Reset Frequency	Number Improved	Ave % Improved	Coverage	Cost	Weighted % Improve
5	50	40	22.61	3472	24.27	24.84
	150	40	21.83	3603	23.59	24.83
	450	39	18.42	3865	23.29	22.93
25	50	40	27.85	3351	21.48	30.89
	150	40	30.78	3004	20.49	35.86
	450	40	37.37	2596	14.75	37.37
75	50	40	28.06	2594	20.36	39.90
	150	40	35.19	2594	15.63	48.27
	450	40	43.90	1596	13.37	61.54

The smallest problem in terms of initial cost is Problem 3. This problem is based on small scale contingencies and only contains fifty possible combat locations. In almost every experimental run for this problem, a solution was found that simultaneously improves both objectives. However, the solutions generated using a higher transfer size result in the highest level of improvement. For Problem 3, the best solution is found using a transfer size of 75 and a reset frequency of 450. This solution achieved a weighted improvement of 61.54% in comparison to the initial solution.

The results for Problem 4 again indicate the difficulty of finding simultaneously improved solutions using a smaller transfer size. Although a transfer size of 5 had as

Table 8

Simultaneous Improvement of Objectives – Problem 4
Initial Solution (Cost-$59.41 million, Coverage- 5385 miles)

Factors				Best Solution Found		
Transfer Size	Reset Frequency	Number Improved	Ave % Improved	Coverage	Cost	Weighted % Improve
5	50	31	.325	5326	58.89	.99
	150	19	.390	5355	58.69	.89
	450	14	.373	5349	59.05	.64
25	50	29	3.327	2946	52.69	28.30
	150	25	1.951	5128	56.63	4.73
	450	15	4.063	3021	55.04	25.63
75	50	34	11.303	2490	51.96	33.15
	150	24	8.946	2168	47.26	40.10
	450	17	17.596	2143	52.74	35.72

many as 31 runs with a simultaneously improved solution, the average amount of improvement is less than one percent. Similarly the best solutions found for this transfer size result in a weighted improvement of the two objectives of less than one percent. However, as the transfer size increases, the average improvement of the solutions also increases. The greatest average improvement is 17.596% for a transfer size of 75 and a reset frequency of 450. In addition, the best solution for problem 4 is found using a transfer size of 75 and a reset frequency of 150, which results in a weighted improvement of the two objectives of 40.10%.

The results of Problem 5 indicate each of the three transfer levels are similarly effective in finding improved solutions in comparison to the initial solution.

Table 9

Simultaneous Improvement of Objectives – Problem 5
Initial Solution (Cost-$47.52 million, Coverage- 5385 miles)

Factors				Best Solution Found		
Transfer Size	Reset Frequency	Number Improved	Ave % Improved	Coverage	Cost	Weighted % Improve
5	50	39	13.515	3486	39.65	25.92
	150	35	6.534	3573	39.73	25.03
	450	25	5.962	3737	40.42	22.77
25	50	35	21.251	2599	42.92	30.70
	150	33	16.930	2729	40.14	32.42
	450	31	16.641	2759	29.31	43.55
75	50	39	25.625	2658	33.47	40.10
	150	37	29.654	2599	27.81	46.60
	450	38	34.084	2625	22.47	51.99

For each experimental cell for this problem, at least 25 of the 40 runs were able to find at least one solution that simultaneously improved both objectives. The largest average improvement for a cell was 34.084% for a transfer size of 75 and a reset frequency of 450. Similarly, the best solution for Problem 5 was found with the same parameter settings and resulted in a weighted improvement of 51.99% in comparison to the initial solution.

The results for the last problem indicate it may be the most difficult for finding simultaneously improved solutions. Both transfer sizes of 5 and 25 had only between ten and twenty runs with a solution that improved both objectives.

Table 10

Simultaneous Improvement of Objectives – Problem 6
Initial Solution (Cost-$91.68 million, Coverage- 5385 miles)

Factors				Best Solution Found		
Transfer Size	Reset Frequency	Number Improved	Ave % Improved	Coverage	Cost	Weighted % Improve
5	50	11	.194	5325	91.58	.61
	150	12	.130	5367	91.43	.31
	450	10	.146	5379	91.24	.30
25	50	19	.995	5140	89.67	3.37
	150	12	.413	5379	89.68	1.15
	450	13	.460	5325	90.94	.96
75	50	15	4.235	2678	87.48	27.43
	150	16	8.761	2266	88.06	30.93
	450	35	31.422	2389	67.38	41.07

In addition, the average improvement for all of the solutions for these two levels was less than one percent in comparison to the initial solution. In addition, the best solution found using a transfer size of 25, only resulted in a simultaneously weighted improvement of 3.37%. The transfer size of 75 far outweighed the results of the other levels in terms of number of runs finding an improved solution and the magnitude of this improvement. More specifically, the experimental cell with a transfer size of 75 and a reset frequency of 450 found an simultaneously improved solution 39 out of 40 runs with an average improvement of 31.422%. In addition, the best solution for problem six was found with these parameter settings resulting in simultaneous weighted improvement of 41.07% in comparison to the initial solution for the problem.

Effect of Problem Number

The second research question asks whether the problem type is a significant factor in determining the cost and coverage of the solutions generated by the model. It was believed prior to the study that this factor would be significant since the demands for each problem differ in the size, location and number of combat locations. The question is analyzed by conducting a three factor Multiple Analysis of Variance (MANOVA) for the best result obtained from each of the forty runs of each experimental cell with equal weighting of the two dependent variables. The results for this MANOVA indicate a significant difference for the problem type with Wilks' $\lambda < 0.023$, $F(10, 4210)=2345.57$, with a corresponding p-value ≤ 0.0001. In addition, the results for this three factor analysis also indicate significant differences for the transfer size (Wilks' $\lambda < 0.366$,

F(4,4210) =687.313), and reset frequency (Wilks' λ <0.850, F(4, 4210)=88.914), both with p-values $\leq$ 0.0001. However, the analysis also indicates that the three way interaction of problem type, reset frequency, and transfer size is significant with Wilks' λ <.742, F(40,4210)=16.952 and a p-value $\leq$ 0.0001. In addition, each of the three two-factor interactions is significant with p-values $\leq$ 0.0001.

Table 11

3-Factor Analysis of Variances (ANOVAs) for Cost and Coverage

Source	Dependent Variable	*df*	*F*	*p*≤
Intercept	Cost	1	258866.26	.0001
	Coverage	1	96868.63	.0001
Problem	Cost	5	13037.06	.0001
	Coverage	5	104.93	.0001
Transfer	Cost	2	233.85	.0001
	Coverage	2	998.63	.0001
Reset	Cost	2	180.08	.0001
	Coverage	2	13.78	.0001
Problem*Transfer*Reset	Cost	20	25.13	.0001
	Coverage	20	17.40	.0001
Problem*Transfer	Cost	10	39.54	.0001
	Coverage	10	29.50	.0001
Problem*Reset	Cost	10	34.81	.0001
	Coverage	10	16.01	.0001
Transfer*Reset	Cost	4	43.83	.0001
	Coverage	4	187.02	.0001
Error	Cost	2106		
	Coverage	2106		

Closer examination of the three-factor ANOVA's for each dependent variable in Table 11 reveals the main effect and interactions are similarly significant for each dependent

variable. Table 12 includes the mean values for cost and coverage for each of the six problems. The results in Tables 11 and 12 appear to indicate a high degree of difference for the cost associated with each problem and a smaller, yet still significant ($F(5)=104.93$, $p \leq .0001$), difference in the coverage values for each problem.

Table 12

Cost and Coverage for Individual Problems

Problem	Cost ($millions)	Coverage (miles)
1	73.70 (10.10)	3818.46 (1194.86)
2	138.73 (13.96)	3099.68 (455.04)
3	22.18 (3.30)	3355.44 (379.95)
4	68.59 (9.19)	3563.31 (1134.96)
5	45.90 (8.37)	3302.48 (693.04)
6	105.80 (10.94)	3135.02 (694.29)

However, additional interpretation and testing of the factors would require further understanding of the nature of the interactions between the three factors and the development of confidence intervals for each of the fifty four treatment cells for both dependent variables. Therefore, further analysis for research questions three and four is conducted using separate two-factor MANOVAs for each of the six problems. In order to account for having several tests, the level of significance for individual tests is adjusted accordingly using the Bonferoni inequality in order to maintain a high level of significance for the entire family of tests.

Transfer Size and Reset Frequency

The third and fourth research questions ask whether or not the transfer size and reset frequency used in the search make a significant difference in determining the cost and coverage of the solutions. It was believed prior to the study that the number of weapons relocated during each inventory transfer would make a significant difference in the quality of the final solutions. In addition, it was believed that how often the search is reset back to the best coverage solution would also play a significant role in finding improved solutions in comparison to the initial solution. In order to analyze these questions, a two factor MANOVA is conducted for each of the six problems on the best solution from each run based on equal weighting of the two dependent variables. The results of the Multivariate tests for all six problems are found in Table 13. The results of these tests are significant for seventeen of the eighteen tests, with an individual significance level of $\alpha = .005$ and a family significance level $\alpha \leq .09$ as determined by the Bonferoni inequality. Only the multivariate test for the reset frequency in Problem 5 appears to be insignificant. The results of these tests justify the exploration of the individual 2-Factor ANOVAs for each dependent variable. This analysis is conducted separately on each problem in order to fully understand the nature of the interactions and applicable main effects for each problem. General comparisons of the results between problems are made at the end of this chapter and again in the Conclusion chapter.

Table 13

Multivariate Tests

Problem	Effect	Wilks' λ	*df*	*F*	$p \leq$
1	Transfer	.319	4/700	134.71	.0001
	Reset	.747	4/700	27.42	.0001
	Transfer*Reset	.455	8/700	42.17	.0001
2	Transfer	.156	4/700	267.86	.0001
	Reset	.375	4/700	110.76	.0001
	Transfer*Reset	.504	8/700	35.69	.0001
3	Transfer	.294	4/700	147.56	.0001
	Reset	.484	4/700	76.65	.0001
	Transfer*Reset	.567	8/700	28.73	.0001
4	Transfer	.353	4/700	119.45	.0001
	Reset	.934	4/700	6.10	.0001
	Transfer*Reset	.705	8/700	16.73	.0001
5	Transfer	.429	4/700	92.20	.0001
	Reset	.966	4/700	3.10	= .0150
	Transfer*Reset	.708	8/700	16.52	.0001
6	Transfer	.243	4/700	179.91	.0001
	Reset	.564	4/700	58.00	.0001
	Transfer*Reset	.476	8/700	39.32	.0001

Problem 1 Analysis. Following a significant multivariate test, the results for the individual ANOVAs is explored for the first problem, the MTW scenario with 50 combat locations. For this analysis, there are two factors and one interaction term for each dependent variable resulting in a family significance level of $\alpha \leq .03$, with $\alpha = .005$ for each individual test. The results of the ANOVA for Problem 1 are listed in Table 14. In this analysis, the interaction term of the two factors is significant for both dependent variables. Therefore, the difference in the dependent variables for each factor is subject to

the corresponding level of the other factor. In order to understand this relationship, the confidence intervals for the individual treatment means are developed using the Bonferoni procedure and multiplier for all potential intervals.

Table 14

2-Factor ANOVAs of Cost and Coverage for Problem 1

Source	Dependent Variable	*df*	*F*	$p \leq$
Intercept	Cost	1	32936.86	.0001
	Coverage	1	11597.05	.0001
Transfer	Cost	2	9.90	.0001
	Coverage	2	205.59	.0001
Reset	Cost	2	30.52	.0001
	Coverage	2	40.78	.0001
Transfer*Reset	Cost	4	46.21	.0001
	Coverage	4	72.17	.0001
Error	Cost	351		
	Coverage	351		

These intervals are used to determine which means are significantly different from the others with $\alpha = .005$ and a family significance level $\alpha \leq .09$ for the eighteen intervals for the problem. The results of this comparison and a relative ranking of the intervals in terms of desirability are listed in Table 15. In analyzing these results, it is difficult to tell if any one transfer size or reset frequency is more efficient at finding improved solutions to the problem. However, some general observations can be made. First, the four highest ranking treatment cells for the cost dependent variable have reset frequencies of either

450 or 150. This seems logical since the search is redirected to the best coverage value fewer times in these cells.

Table 15

Treatment Mean Ranks and Intervals for Problem 1

DV	Cost			Coverage		
Rank	Transfer Size	Reset Freq.	Interval	Transfer Size	Reset Freq.	Interval
1	5	450	(61.067, 67.950)	75	450	(2263.36, 2864.35)
2	25	150	(61.610, 68.494	75	150	(2435.04, 3036.04)
3	75	450	(64.497, 71.380)	25	450	(2941.74, 3542.73)
4	5	150	(67.464, 74.347)	75	50	(2975.75, 3576.74)
5	75	50	(70.766, 77.649)	25	50	(3254.38, 3855.37)
6	75	150	(73.230, 80.114)	5	50	(3303.69, 3904.68)
7	25	50	(76.251, 83.134)	5	150	(4479.67, 5080.67)
8	5	50	(77.059, 83.942)	25	150	(4926.20, 5527.19)
9	25	450	(80.399, 87.282)	5	450	(5081.79, 5682.78)

Next, three out of the top four cells for the coverage dependent variable are found using a transfer size of 75. More surprisingly, the three most desirable combinations for the Cover dependent variable are also found using reset frequencies of either 150 or 450. Finally, it should be noted that many of the confidence intervals overlap with $\alpha = .005$ and individual differences between the treatment means is therefore considered insignificant. However, for the Cost dependent variable, the highest ranking combination (5,450) is significant in comparison to the treatment ranked 5-9, and for the coverage

dependent variable, the highest ranking combination (75,450) is significant in comparison to each of the other treatment cells ranked 3-9. Following, the analysis of each problem, the aggregate ranking of the treatment cells for all of the six problems in the study will be considered.

Problem 2 Analysis. Problem 2 is based on the occurrence of Major Theater Wars (MTW) with 100 combat locations and is the largest problem in terms of cost for the initial solution. In Table 13, the multivariate tests for Problem 2 are significant for each of the two factors and the interaction term with $\alpha = .005$ and $p \leq .0001$ for each of the three individual tests. Following the significant multivariate tests, the individual 2-Factor ANOVAs are investigated in order to determine if significant differences are present for the different levels of transfer size and reset frequency used in the search for improved values of cost and coverage. The results of the ANOVAs for Problem 2 are displayed in Table 16. The results for the two factors and interaction term are significant with $p \leq .0001$ for each dependent variable using a family significance level of $\alpha \leq .03$ and $\alpha = .005$ for each individual test. Similar to Problem 1, the interaction term of the two factors is significant for both dependent variables. Therefore, the difference in the dependent variables for each factor is subject to the corresponding level of the other factor, and the confidence intervals for the individual treatment means are developed using the Bonferoni procedure. These intervals determine which means are significantly different from the others with $\alpha = .005$ and a family significance level $\alpha \leq .09$.

Table 16

2-Factor ANOVAs of Cost and Coverage for Problem 2

Source	Dependent Variable	*df*	*F*	*p* ≤
Intercept	Cost	1	114927.45	.0001
	Coverage	1	105644.22	.0001
Transfer	Cost	2	166.92	.0001
	Coverage	2	806.32	.0001
Reset	Cost	2	128.96	.0001
	Coverage	2	37.25	.0001
Transfer*Reset	Cost	4	54.63	.0001
	Coverage	4	58.06	.0001
Error	Cost	351		
	Coverage	351		

The results of the comparison and the ranking of the intervals in terms of desirability are listed in Table 17. Analysis of the results from Table 17 indicate a clearer picture of which combinations of transfer size and reset frequency are dominant in finding improved solutions to the initial solution for the problem. For instance, the treatment cell with a reset frequency of 450 and a transfer size of 75 is able to find significant results in comparison to all other combinations for the cost dependent variable and is the highest ranking in terms of coverage. In addition, a transfer size of 75 is used in the top three highest ranking cells for the coverage dependent variable and for the top two highest ranking cells for Cost. Additionally, a transfer size of 5 is used in all three of the lowest ranking treatment cells for the dependent variable Coverage. The reset frequency of 450 is effective when used with a larger transfer size; however, when combined with a

transfer size of 5 it results in the lowest ranking combination for both dependent variables.

Table 17

Treatment Mean Ranks and Intervals for Problem 2

DV	Cost			Coverage		
Rank	Transfer Size	Reset Freq.	Interval	Transfer Size	Reset Freq.	Interval
1	75	450	(107.701, 114.637)	75	450	(2567.89, 2729.53)
2	75	150	(127.010, 133.945)	75	150	(2599.90, 2761.54)
3	25	450	(129.377, 136.313)	75	50	(2741.53, 2903.17)
4	25	150	(134.789, 141.724)	25	150	(2829.10, 2990.74)
5	75	50	(142.312, 149.248)	25	50	(2880.38, 3042.02)
6	5	50	(143.391, 150.327)	25	450	(2924.32, 3085.96)
7	5	150	(143.653, 150.588)	5	50	(3249.92, 3411.56)
8	25	50	(144.376, 151.311)	5	150	(3463.29, 3624.93)
9	5	450	(144.712, 151.648)	5	450	(3913.39, 4075.03)

Problem 3 Analysis. Problem 3 is a Small Scale Contingency problem with 50 combat locations and is the smallest problem in terms of cost for the initial solution. The results for the multivariate tests for Problem 3 are significant for each of the two factors and the interaction term with $\alpha = .005$ and $p \leq .0001$ for each of the individual tests. Significant multivariate tests justify the investigation of the individual 2-Factor ANOVAs in order to determine if significant differences are present for the different levels of

transfer size and reset frequency used by the search program to find improved solutions to the problem. The results of the ANOVAs for Problem 2 are displayed in Table 18.

Table 18

2-Factor ANOVAs of Cost and Coverage for Problem 3

Source	Dependent Variable	*df*	*F*	$p\leq$
Intercept	Cost	1	67266.28	.0001
	Coverage	1	80155.29	.0001
Transfer	Cost	2	324.99	.0001
	Coverage	2	234.22	.0001
Reset	Cost	2	148.30	.0001
	Coverage	2	4.30	.0140
Transfer*Reset	Cost	4	46.08	.0001
	Coverage	4	49.21	.0001
Error	Cost	351		
	Coverage	351		

Again, the results for the ANOVA interaction terms for each dependent variable are significant with $p \leq .0001$ using a family significance level of $\alpha \leq .03$ and $\alpha = .005$ for each individual test. Therefore, the confidence intervals for the individual treatment means are developed using the Bonferoni procedure. These intervals are used to determine the significance of the mean differences with $\alpha = .005$ and a family significance level $\alpha \leq .09$. The results of the comparison and the ranking of the intervals in terms of desirability are listed in Table 19. The results of the transfer size and reset frequency treatment cells for Problem 3 show consistent results in comparison to the first two problems.

Table 19

Treatment Mean Ranks and Intervals for Problem 3

DV	Cost			Coverage		
Rank	Transfer Size	Reset Freq.	Interval	Transfer Size	Reset Freq.	Interval
1	75	450	(16.152, 17.602)	75	450	(2726.23, 2927.11)
2	25	450	(17.684, 19.134)	75	150	(2924.03, 3124.91)
3	75	150	(20.040, 21.489)	25	450	(3138.45, 3339.33)
4	25	150	(20.899, 22.349)	25	150	(3235.23, 3436.11)
5	75	50	(22.373, 23.823)	75	50	(3248.11, 3488.99)
6	25	50	(22.458, 23.907)	25	50	(3254.57, 3455.45)
7	5	150	(24.394, 25.844)	5	50	(3405.79, 3606.67)
8	5	50	(24.410, 25.860)	5	150	(3504.48, 3705.35)
9	5	450	(24.708, 26.158)	5	450	(3858.14, 4059.02)

The combination of 450 for reset frequency and 75 for transfer size is the highest ranking for both dependent variables. The mean interval for this treatment is significant in comparison to the other eight treatments for the Cost dependent variable and is significant in comparison to all except one of the other treatments for Coverage with $\alpha = .005$. Transfer sizes of 75 and 25 are able to achieve improved solutions in comparison to combinations using a transfer size of 5. This relationship is the clearest for the Cost dependent variable where all combinations using a transfer size of 25 and 75 are significant in comparison to the three cells using a transfer size of 5. The lowest results for the problem are once again found using the treatment with transfer size 5 and reset frequency 450. The reset frequency factor appears to be the least significant; especially

for the Coverage dependent variable where the main effect is not significant with $p \leq .014$.

Problem 4 Analysis. Problem 4 is a Small Scale Contingency problem with 100 combat locations and is the second smallest problem in terms of cost for the initial solution. From Table 13, the results for the multivariate tests for Problem 4 are significant for each of the two factors and the interaction term with $\alpha = .005$ and $p \leq .0001$ for each of the three individual tests. The results of the ANOVAs for Problem 4 are displayed in Table 20. In Table 20, the results of the interaction terms for each dependent variable are significant with $p \leq .0001$ using a family significance level of $\alpha \leq .03$ and $\alpha = .005$. In addition, the main effect of reset frequency for the Coverage dependent variable is insignificant with $p \leq .7840$.

Table 20

2-Factor ANOVAs of Cost and Coverage for Problem 4

Source	Dependent Variable	*df*	*F*	$p\leq$
Intercept	Cost	1	27832.10	.0001
	Coverage	1	7277.08	.0001
Transfer	Cost	2	38.55	.0001
	Coverage	2	134.54	.0001
Reset	Cost	2	9.00	.0001
	Coverage	2	.243	= .7840
Transfer*Reset	Cost	4	12.93	.0001
	Coverage	4	28.91	.0001
Error	Cost	351		
	Coverage	351		

In order to understand the nature of the interactions between the two factors, the confidence intervals for the individual treatment means are created using the Bonferoni procedure. These intervals determine the significance of the mean differences with $\alpha =$ .005 and a family significance level $\alpha \leq .09$. The ranking of the intervals in terms of desirability are listed in Table 21 and the results for this problem contain differences to previous problems worth noting. First, the combination of transfer size 5 and reset frequency 450 achieves the highest ranking in terms of the Cost dependent variable despite the fact that it had been a poor choice for previous problems. Although, this combination did achieve the highest ranking it should be noted that its interval is not significantly different than the next three highest ranking combinations.

Table 21

Treatment Mean Ranks and Intervals for Problem 4

DV	Cost			Coverage		
Rank	Transfer Size	Reset Freq.	Interval	Transfer Size	Reset Freq.	Interval
1	5	450	(57.240, 64.209)	75	450	(1969.83, 2677.82)
2	75	50	(59.983, 66.952)	75	150	(2599.09, 3307.08)
3	75	450	(61.054, 68.023)	25	150	(2770.75, 3478.74)
4	5	150	(63.063, 70.032)	25	450	(2877.80, 3585.79)
5	75	150	(65.652, 72.620)	75	50	(2898.05, 3606.04)
6	25	50	(66.280, 73.249)	5	50	(3358.29, 4066.28)
7	5	50	(68.384, 75.352)	25	50	(3375.84, 4083.84)
8	25	450	(70.987, 77.955)	5	150	(4149.00, 4856.99)
9	25	150	(73.331, 80.300)	5	450	(4885.16, 5593.15)

In addition, the corresponding Coverage interval for this treatment cell is the lowest ranking of the nine treatments and is significantly inferior to all other treatments for the Coverage dependent variable. Therefore, it is believed that while this treatment was able to achieve its best solutions with low costs, it did so only while achieving poor coverage values. A similar result for this combination is seen in the results for Problem 1 where an improved Cost value is achieved, while the Coverage value for the treatment is the worst found by any combination in the study. Next, unlike the previous two problems, the overlapping of confidence intervals for Problem 4 is quite prevalent for the Cost dependent variable making it difficult to determine if any one treatment is clearly superior in finding improved solutions to the problem. However, the results for the Coverage dependent variable for Problem 4 are consistent with previous problems and the combination of transfer size 75 and reset frequency 450 is significantly lower than seven of the eight other treatment cells. Similarly, a transfer size of 5 appears to be lowest ranking level in terms of obtaining improved Coverage values for Problem 4.

Problem 5 Analysis. Problem 5 contains 50 combat locations and is a combination of Major Theater War scenarios and Small Scale Contingencies. The results for Problem 5 are different in comparison to each of the other five problems in the study resulting in slightly different presentation of results. First, only two of three multivariate tests for problem 5 are significant. The main factor effect for Transfer size and the interaction term of the two factors are significant with $\alpha = .005$ and $p \leq .0001$. However, the multivariate test for the reset frequency is insignificant with $p \leq .015$ and $\alpha = .005$. The results of the individual ANOVAs for Problem 5 are displayed in Table 22. Analysis

of the interaction terms results in a significant outcome for the Coverage dependent variable with $p \leq .0001$ using a family significance level of $\alpha \leq .03$ and $\alpha = .005$. The individual treatment differences for this dependent variable are explored using the

Table 22

2-Factor ANOVAs of Cost and Coverage for Problem 5

Source	Dependent Variable	*df*	*F*	$p \leq$
Intercept	Cost	1	13983.32	.0001
	Coverage	1	17325.51	.0001
Transfer	Cost	2	47.87	.0001
	Coverage	2	141.75	.0001
Reset	Cost	2	2.99	= .0510
	Coverage	2	3.63	= .0280
Transfer*Reset	Cost	4	2.61	= .0350
	Coverage	4	29.79	.0001
Error	Cost	351		
	Coverage	351		

Bonferoni procedure similar to previous problems. However, the interaction term for the Cost dependent variable is insignificant with $p \leq .0350$ and $\alpha = .005$. Therefore, the main effects are analyzed for the Cost dependent variable. The reset frequency main effect is insignificant with $p \leq .0510$ and $\alpha = .005$, therefore no further analysis is required for this factor. However, the transfer size main effect is significant with $p \leq .0001$ and $\alpha = .005$. Tukey's pairwise comparisons are displayed in Table 23 for the transfer size differences for Problem 5 using $\alpha = .02$ and a family $\alpha = .06$ for the three confidence

intervals. These comparisons show that a transfer size of 75 is able to obtain costs between \$4.558 million and \$9.658 million lower than a transfer size of 25, regardless of which reset frequency is used to solve Problem 5.

Table 23

Tukey Pairwise Comparisons of Cost (\$million) for Transfer Size in Problem 5

Transfer Size 1	Transfer Size 2	Mean Difference	Interval of Mean Difference	p-value ≤
5	25	1.644	(-9.065, 4.194)	= .1940
5	75	8.752	(6.202, 11.302)	.0001
25	75	7.108	(4.558, 9.658)	.0001

Also, a transfer size of 75 is able to find costs between \$6.202 million and \$11.302 million lower than a transfer size of 5 for any reset frequency used to find solutions to Problem 5. No significant difference is evident between transfer sizes 5 and 25.

The individual treatment means and their confidence intervals as determined by the Bonferoni procedure are displayed for the Coverage dependent variable in Table 24. The results of the treatment mean analysis for the Coverage dependent variable for Problem 5 are consistent with the analysis for previous problems. The intervals are determined with $\alpha = .005$ for each individual interval and the combination of 75 for the transfer size and 450 for the reset frequency is again the highest ranking treatment.

Table 24

Treatment Mean Ranks and Intervals of Coverage for Problem 5

Rank	Transfer Size	Reset Frequency	Interval
1	75	450	(2307.02, 2732.28)
2	75	150	(2587.68, 3012.94)
3	25	50	(2941.55, 3366.80)
4	75	50	(2953.80, 3379.06)
5	25	450	(3035.15, 3247.78)
6	25	150	(3057.50, 3482.76)
7	5	50	(3274.01, 3699.27
8	5	150	(3441.85, 3867.10)
9	5	450	(4210.06, 4635.31)

This combination is significant in comparison to seven of the eight other possible treatments used for finding solutions to the problem. In addition, the transfer size of 5 is used in the three least preferred combinations and the combination of 5 for transfer size and 450 for reset frequency is the least favorable combination and is significantly higher than each of the other eight combinations with $\alpha = .005$ and the family significance level $\alpha = .09$.

Problem 6 Analysis. Problem 6 includes 100 combat locations and is a combination of MTW and SSC scenarios. From Table 13, the results for the multivariate tests for Problem 6 are significant for each of the two factors and the interaction term with $\alpha = .005$ and $p \leq .0001$ for each of the individual tests. The results of the ANOVAs for Problem 6 are displayed in Table 25. The interaction terms for each dependent

variable are significant with $p \leq .0001$ using a family significance level of $\alpha \leq .03$ and $\alpha = .005$.

Table 25

2-Factor ANOVAs of Cost and Coverage for Problem 6

Source	Dependent Variable	*df*	*F*	$p \leq$
Intercept	Cost	1	79958.88	.0001
	Coverage	1	25837.60	.0001
Transfer	Cost	2	84.35	.0001
	Coverage	2	320.43	.0001
Reset	Cost	2	124.99	.0001
	Coverage	2	22.03	.0001
Transfer*Reset	Cost	4	20.68	.0001
	Coverage	4	56.95	.0001
Error	Cost	351		
	Coverage	351		

In order to understand the interaction between the two factors, the confidence intervals for the individual treatment means are developed using the Bonferoni procedure. These intervals determine the significance of the mean differences with $\alpha = .005$ and a family significance level $\alpha \leq .09$. The intervals are ranked in terms of desirability in Table 26. The results of the treatment intervals are consistent with previous problems, and the combination of transfer size 75 and reset frequency 450 is the highest ranking for both dependent variables. This treatment is significant in comparison to all other combinations for the Cost dependent variable and is significant in comparison to all

except one other treatment for the Coverage dependent variable with $\alpha = .005$ for each interval.

Table 26

Treatment Mean Ranks and Intervals for Problem 6

DV	Cost			Coverage		
Rank	Transfer Size	Reset Freq.	Interval	Transfer Size	Reset Freq.	Interval
1	75	450	(81.263, 87.604)	75	450	(2298.78, 2629.35)
2	75	150	(100.268, 106.609)	75	150	(2392.21, 2722.79)
3	5	450	(101.087, 107.429)	25	50	(2723.88, 3054.46)
4	25	450	(101.766, 108.118)	25	150	(2744.15, 3074.72)
5	25	150	(104.810, 111.152)	75	50	(2799.72, 3130.30)
6	75	50	(105.729, 112.071)	25	450	(2815.55, 3311.41)
7	5	150	(107.652, 113.994)	5	50	(3125.51, 3456.09)
8	5	50	(109.468, 115.809)	5	150	(3483.92, 3814.49)
9	25	50	(111.577, 117.919)	5	450	(4343.89, 4674.47)

In addition, for the Coverage dependent variable the transfer size of 5 is present in the three lowest ranking combinations and a reset frequency of 450 and transfer size of 5 obtains a coverage value that is significantly less desirable than all other combinations.

Comparison of Treatment for All Problems. Further understanding of the ability of each combination of transfer size and reset frequency to obtain improved solutions for the six problems is explored by creating an aggregate ranking of the combinations across the six problems. Although some information is admittedly lost in

this non-parametric analysis, this method gives the average ability of the treatment to obtain the most desirable results in terms of cost and coverage for all six problems. It is intended to aid in the analysis of how desirable each combination is in obtaining improved solutions for both objectives. The results of the aggregate ranking are listed in Table 27 and several trends seen in individual problems are confirmed through this analysis. First, the combination of transfer size 75 and 450 is clearly dominant in comparison to the other treatments. This combination achieves an aggregate ranking of 1.67 for the Cost variable and is the top selection in all six problems for coverage.

Table 27

Average Rank of Treatment Means for all Problems

Rank	Transfer Size	Reset Frequency	Cost Ave Rank	Coverage Ave Rank
1	75	450	1.67	1.00
2	75	150	3.33	2.00
3	75	50	4.33	4.33
4	25	450	5.33	4.50
5	25	150	5.17	4.83
6	25	50	6.67	4.83
7	5	450	5.17	9.00
8	5	150	6.33	7.83
9	5	50	7.00	6.67

In addition, the other two combinations using a transfer size of 75 achieve the next highest rankings for both the Cost and Coverage variables. This separation between transfer sizes continues for the Coverage dependent variable and the three combinations

using a transfer size of 25 achieves average ranks below those with 75 and above those using 5. This clear separation in ranks is not as evident for different levels of reset frequency or for the lower ranking cost combinations. Continued comparison of all of the results from this chapter as they relate to the research questions for the study is continued in the Conclusions chapter.

Conclusions

Discussion of the results as they pertain to each individual research question is presented in addition to some general conclusions observed during the study. In addition, direction for further research relating to the US Air Force munitions pre-positioning problem is presented.

Simultaneous Improvement of Initial Solutions

The results indicate that for each of the six problems the search program is able to find solutions that simultaneously improve Cost and Coverage in comparison to the values for these two dependent variables obtained by the initial solution. However, the ability of the program to find improved solutions and the quality of the solutions varies between different problems and between the different transfer sizes and reset frequencies used in the searches. For instance, the number of runs which attain an improved solution differs greatly from one problem to the next. In the smallest problems which contain only 50 combat locations (Problems 1,3, and 5), the number of runs with at least one solution that simultaneously improves both dependent variables is higher in comparison to the three problems with 100 combat locations. For example in Problem 5, the number of simultaneously improved solutions ranges from 25 to 39, and in its 100 combat location counterpart, Problem 6, the number of improved solutions only ranges from 10 to 35. Similarly, the number of improved solutions is higher for problems where the

initial solution cost is low. For example, in comparing the three problems with 50 combat locations the number of improved solutions is higher for problems with a low initial cost, and the search algorithm is able to find at least one simultaneously improved solution for 359 out of 360 runs for the smallest problem, Problem 3. Additionally, throughout the six problems analyzed, the number of simultaneously improved solutions also varies depending on the value of the transfer size and reset frequency parameters within the search. The transfer size of 75 is dominant throughout the six problems. With only a few exceptions, the average percentage improvement of solutions found with a transfer size of 75 is better than the same percentage found with any other transfer size. In addition, the best solution for each problem is found using a transfer size of 75 and these solutions range in improvement from 37.70% for Problem 2 to 61.54% for Problem 3. Similarly, the reset size plays a role in finding simultaneously improved solutions. The unique setting of 75 for transfer size and 450 for reset frequency finds the best solution in terms of equally weighted dependent variables for five out of the six problems. In addition, this combination also finds the highest average percentage improvement for simultaneously improved solutions for all six problems in the study.

Within the results, it is clear that solutions to each of the six problems are able to be obtained that simultaneously improve both Cost and Coverage with as much as 61.54% improvement in comparison to the initial solution. However, the ability of the program to find such improvements appears to depend on how many combat locations are contained in a problem and the size of the initial cost of the problem. Regardless of the problem number, using a transfer size of 75 and a corresponding reset frequency of

450 appears to lead to finding the best simultaneous improved solutions in comparison to the initial solution. These results together lead to a positive conclusion for Research Question 1, and simultaneously improved solutions in comparison to the initial solution are found using the search algorithm developed for the study.

Problem Configuration

The three factor MANOVA of Cost and Coverage of the best solutions in terms of equal weighting revealed significant results for the three main effects of problem number, transfer size and reset frequency. However, since the three-way and two-way interactions were also significant; the pairwise differences for the main effects were not explored. Instead the remainder of the analysis was divided into six separate two-factor MANOVAs in order to understand the main effects and interaction of transfer size and reset frequency separately for each individual problem. The mean values of cost and coverage for each problem provided Table 12 show that there are significant differences in the best cost solutions found by the algorithm for the six problems. It was originally thought that since the initial coverage value of each problem was identical that the final best coverage values might not differ significantly. However, the results of the three factor MANOVA did show significant differences in the coverage values with $F(5)=104.93$ and $p \leq .0001$, adding to the conclusion that there are differences in the results dictated by the different size and configurations of the six problems. Finally, comparison of the mean cost values and coverage values indicate other differences in the results for the six problems. Large problems with 100 combat locations and high initial

costs (Problems 1, 2, 4 and 6) make most of their improvement in the coverage variable and maintain a mean cost near or above the initial cost for the problem. Therefore for these four problems, it appears that the best values find much of their improvement in terms of coverage, even though individual solutions can be found that dramatically reduce both solutions as seen in the results for Research Question 1. In contrast, Problems 3 and 5 have mean cost and coverage values from their best weighted solutions that are both smaller than those of the initial solution.

Finally, based on the results of the three-factor MANOVA, the results of the individual ANOVAs for cost and coverage, and the comparison of the mean values in Table 12, a positive result is concluded for Research Question 2. The problem size and configuration does make a significant difference in the best cost and coverage values able to be achieved by the search algorithm used in the study. The remaining analysis and results for Research Question 3 and 4 are conducted based on the positive result for this research question and are analyzed using separate 2-Factor MANOVAs for each question.

Transfer Size

The results indicate that for each problem the transfer size used by the search algorithm makes a significant difference in the quality of the solutions as measured by the dependent variables, Cost and Coverage. However, the impact of transfer size differs between the problems and the reset frequencies, and is at times much clearer for the Coverage dependent variable.

The most evident result for transfer size is seen in the Cost dependent variable for Problem 5, where the main effect for transfer size is significant with no interaction effect with the reset frequency. In this problem, the Cost of solutions found with a transfer size of 75 is $11.30 million to $6.20 million smaller than those found using a transfer size of 5, and $9.66 million to $4.56 million smaller than those found using a transfer size of 25. The significance of transfer size is not as apparent for the Cost dependent variable for problems 1 and 4 and no one transfer size appears to be superior in comparison to the others for these two problems. However, for problems 2, 3 and 6, the larger transfer size of 75 again finds significantly improved solutions in comparison to the other transfer sizes. In problems 2 and 6, the combination of 75 for transfer size and 450 for reset frequency finds Cost values significantly lower than each of the other eight transfer size and reset frequency combinations.

The transfer size of 75 also seems to find improved results in comparison to the other two transfer sizes for the Coverage variable. In Table 15 for problem 1, the transfer size of 75 is used in three of the top four ranking combinations for Coverage and the combination of 75 for transfer size and 450 for reset frequency is significant in comparison to seven of the remaining eight combinations for Coverage values. In addition, for each of the other five problems, the combination of 75 and 450 is significant in comparison to seven of the eight other combinations and only the combination of 75 and 150 is able to find Coverage values that compete with this treatment. The transfer size of 75 is not as favorable when combined with a reset frequency of 50; however for

all six problems it is clearly preferred for finding lower Coverage values when using the highest reset value.

The comparison of the aggregate ranks of the different combinations seen in Table 29 again highlights the difference made by transfer size. For cost, the three highest ranking combinations each use a transfer size of 75 and similarly for Coverage the highest three ranking combinations use 75 as the transfer size. Additionally, for the coverage variable, each combination using a transfer size of 25 is higher ranking than those using a transfer size of 5. Finally, based on the results of the analysis of each independent variable and the results of the aggregate ranking of each individual problem, a positive conclusion for Research Question 3 is concluded. The transfer size used by the search algorithm does make a significant difference in the outcome of the cost and coverage values for solutions to the munitions pre-positioning problem.

The ability of the algorithm to find improved solutions appears to strengthen as the transfer size is increased. Therefore, a subsequent question is whether or not the improvement will continue outside of the range of transfer sizes chosen for the experiment in this study. In order to understand this question, additional testing of the model reveals that the marginal benefit of increasing the transfer size rapidly declines for values larger than 100. This relationship is depicted in Figure 8, where several larger transfer sizes are used in finding solutions to problem 5 with a reset frequency of 450. These results are consistent with pilot-testing conducted prior to the study which indicated that the largest marginal improvements in the objective function values occurred when the transfer size was increased from 1 to 75. The preferred transfer size

for finding solutions to similar pre-positioning problems is believed to be related to the size of the problem as measured by the amount of inventory and number of locations being modeled, and is an area open for further research.

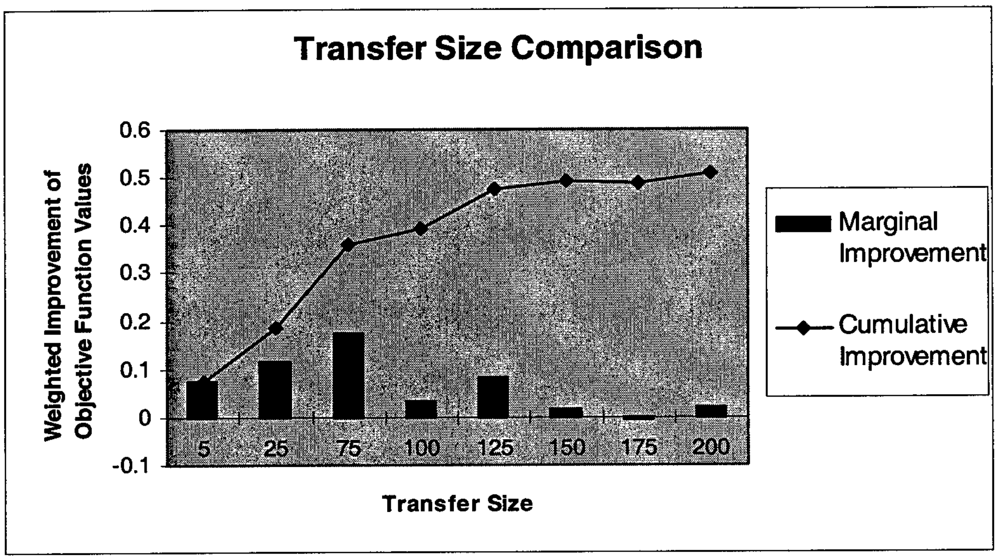

Figure 8. Comparison of Larger Transfer Sizes

The results of the study appear to indicate that higher transfer sizes are not only able to find improved Cost values for the munitions pre-positioning problem, but they also seem to play a significant role in finding improved Coverage values. Prior to the study, it was speculated that reset frequency would play a significant role in improving Coverage values, and that transfer size would primarily affect the final cost of solutions found by the search algorithm. However, the results of the study indicate that transfer size may play an even more important role in finding improved Coverage values. It is believed that moving larger number of munitions in each transfer allows the algorithm to

more quickly reposition munitions to newly opened locations in the search process. Therefore, munitions are more quickly moved to locations which improve the distances related to the Coverage dependent variable. The relationship between transfer size and the opening of new locations is a subject to be explored in future research.

Reset Frequency

The results indicate that the reset frequency used by the search algorithm may make a significant difference in the quality of the solutions as measured by the dependent variables, Cost and Coverage. In addition, influence of reset frequency changes between problems and is much clearer for the Coverage dependent variable. For Coverage, significant and consistent results are evident in each of the six problems; however, for the Cost dependent variable only three of the six problems show consistent results and the impact of reset frequency is unclear in the remaining three.

The reset frequency in the search algorithm redirects the search back to the best solution in terms of coverage, and it is therefore logical the variable has a significant impact on the quality of Coverage values found by the search. The first important result is that within the transfer size of 75, the reset frequency 450 finds significantly lower coverage values in comparison to the reset frequency of 50 for each of the six problems. In addition, the combination of 75 and 150 is the second highest ranking in the aggregate rankings of Table 27 for the Coverage variable, although the differences between this combination and 75 and 450 are not always significant. These results appear to indicate that to find the best results using a transfer size of 75, the search should be allowed to

proceed for a larger number of iterations before resetting to the best coverage solution. Resetting after 50 iterations actually appears to frustrate the search algorithm's ability to find improved solutions in terms of coverage and is probably preventing the algorithms ability to explore better solutions prior to resetting. Although, the transfer size of 5 has been shown to be inferior in comparison to higher transfer sizes, it is interesting to note that the effect of reset frequency is not the same when using this transfer size. For example, in each of the six problems the reset frequency of 50 finds significantly improved solutions in comparison to 450 when using a transfer size of 5. This result is also evident in Table 27 where the lowest aggregate ranking for Coverage is achieved by the combination of transfer size 5 and reset frequency 450. However, in analyzing this result it should be remembered that the results using a transfer size of 5 are typically inferior to other transfer sizes by thousands of miles and at times do not even make significant improvement over the coverage value of the initial solution. Therefore, the most important result for the coverage dependent variable should be considered the significant differences seen in the preferred transfer size of 75, where the higher reset frequency of 450 achieves improved solutions over lower values.

The impact of reset frequency on the Cost dependent variable is more difficult to observe and does not appear to be as significant. For example, in Problems 1, 4 and 5, no obvious pattern of significant differences between reset frequencies is evident within the different transfer sizes. This finding is the most obvious in Problem 5 where the main effect for reset frequency is insignificant ($p \leq .051$ and $\alpha = .005$), and the interaction between reset frequency and transfer size is insignificant. Additionally, very few of the

treatment mean intervals for problem 1 and 4 are significantly different from each other making it impossible to identify any pattern of differences between the reset frequency sizes. However, for problem 2, 3 and 6, a pattern of significant differences in reset frequencies is visible. For each of these problems, the reset frequency of 450 is significant in comparison to 150 and 50 for transfer size 75. Additionally, reset frequency 450 is significant in comparison to 50 for transfer size 25 for each of the six problems. These differences for these three problems indicate results consistent with the preferred reset frequency and transfer size combinations for the coverage variable and provide some evidence about which reset frequency to use in the search algorithm.

The conclusion for the fourth research question is positive, and it can be said that differences do exist in the Cost and Coverage values of solutions to the munitions pre-positioning problem based on the reset frequency used in the search. However, this conclusion is the most difficult to make out of the four research questions and is more evident for the Coverage dependent variable where significant differences for the reset frequency were apparent in each of the six problems. For the Cost dependent variable, it should be said that this conclusion is only apparent for three of the six problems. Additionally, for different transfer sizes the direction of the significant differences created by the reset frequency may be different, as seen in the Cost differences for the transfer size of 5. In conclusion, evidence is visible that the higher reset frequency of 450 is able to find improved Cost and Coverage solutions when paired with the larger transfer sizes of 75 or 25. However, the occurrence and size of this improvement will differ depending on the problem configuration and size being solved.

General Conclusions

In addition to conclusions related directly to the four research questions, additional conclusions about the performance and usefulness of the algorithm are made based on experience gained during the experimental study.

Analysis of Opened and Closed Warehouses. In finding solutions to each of the individual problems, the search algorithm explored various combinations of potential warehouse locations by iteratively opening and closing current warehouse locations. For each experimental cell in the study, forty runs were performed and quite often an individual run found a slightly different combination of warehouse locations to use in the best solution in terms of Cost and Coverage. Therefore, by analyzing the final warehouse locations selected during each experimental run, the decision maker can gain insight into which locations might be the most suitable for construction of a new warehouse. Such information could be especially significant if the decision maker is limited to opening only one or two of the possible locations. For example, in the forty experimental runs for Problem 5 using a transfer size of 75 and a reset frequency of 450, a variety of location combinations are used to find the best solution. However, patterns do exist and some initially closed locations are opened repeatedly from one experimental run to the next as seen in Figure 9. In this figure, it can be seen that locations 18 and 79 (Minsk and Baguio) are opened from a closed state in approximately one-third of the runs.

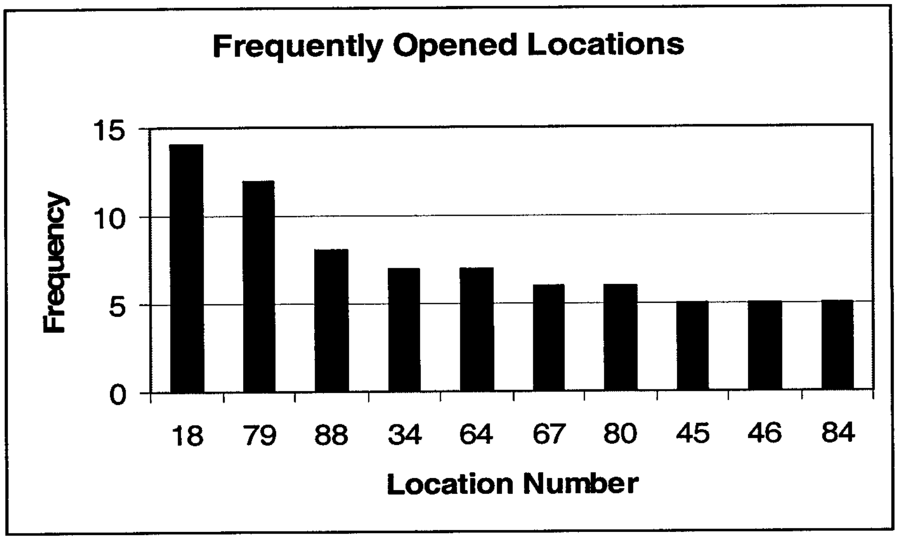

Figure 9. Locations Opened in Problem 5

Additional analysis, including each of the six problem types has the potential to indicate which locations are opened regardless of the demand scenario and therefore provide flexibility for the uncertain future. In addition, the algorithm's ability to identify and save inventory quantities provides insight on what quantities to store in a newly opened warehouse location once the decision has been made to open the location. Experimental analysis of the best locations to open and what quantities to store at these locations is a subject for future research and will benefit from using the best transfer size and reset frequency combinations found in the results of this study.

Relationship between Cost and Coverage. In many of the experimental runs, the search program appears to be able to improve the initial solution by gaining large improvements in the coverage dependent variable while maintaining a final cost at or near the initial cost. Similarly, in the efficient frontiers of the problems such as Figure 6,

it can be seen that there are many alternate solutions in comparison to the best solution that have either the same coverage value achieved at a higher cost, or a similar cost value achieved with higher coverage. Originally, it was thought that a simple linear relationship might be evident in the efficient frontiers and that to accomplish a significant increase in coverage a dramatic increase in cost would be necessary. However, to understand the relationship between cost and coverage it must be remembered that the total cost values seen in the efficient frontiers include both transportation costs and the costs of opening and closing new warehouse locations. Therefore, as coverage is dramatically reduced by opening several new warehouse locations the total cost of the solution also increases due to the costs of building new warehouses. However, these opening costs are offset by similarly dramatic decreases in transportation costs resulting from the smaller coverage distances from supply to demand locations. Therefore, solutions with different coverage values may have the same total cost in an efficient frontier, but differences do exist in the transportation cost and warehouse costs for these solutions. A superior solution is achieved when the transportation costs are reduced by an amount far greater than the necessary costs to open and close warehouses for the solution. The result is a solution which simultaneously reduces both objectives of cost and coverage and is a point to the far bottom left of the efficient frontiers as they are depicted in this study.

Future Research

In addition to the areas previously mentioned, several additional topics warrant further research. First, the search algorithm for finding improved solutions to the munitions pre-positioning problem in this study is constructed in such a manner as to allow for sensitivity analysis of the transportation costs for different regions of the world and the availability of different modes of transportation. This allows the decision maker to understand and test the effect of changes in these variables on the final pre-positioning of munitions inventories. For example, certain regions may be restricted to only using certain modes of transportation or the costs of transporting munitions may be changed for a mode of transportation such as air transportation. Making transportation costs and mode availabilities stochastic inputs to the model would further increase the validity of the model and improve its ability to find the most robust pre-positioning of munitions. Future research should continue to analyze US Air Force data in order to find reasonable distributions of the costs and availability of transportation assets in the different regions of the world in order to create stochastic inputs to the model and find improved solutions to the munitions pre-positioning problem.

Next, the demand for munitions needed in future conflicts is highly uncertain and can vary greatly depending on the nature of the conflict. Future research by the US Air Force should look to improve on the demand set used in this study. The six problems for the study included either 50 or 100 combat locations and were based on three different scenarios based on the occurrence of major theater wars and small scale contingencies. The use of actual intelligence data for the prediction of future conflicts would greatly

enhance the accuracy and validity of the results of further studies. However, it is believed that the use of such data might also render the results of such a study "For Official Use Only" by the US Air Force and therefore all data for this study has been maintained at a purely notional level.

Finally, improvement of the search algorithm itself is possible with the inclusion of a routing subroutine that would identify the actual route that individual munitions are moved from supply warehouses to demand locations for each new solution generated. Such a subroutine would eliminate the need for the circuitous routing factors used by this research to simulate transportation routing and would be similar to the routing routines seen in Location Routing Problems (LRP) previously studied by Chan et al. (2001), Tuzun & Burke (1999), and Srivastava (1993). The inclusion of routing in the problem would change the composite problem in this study to a combined resource allocation and LRP. The formulation of such a problem and modification of the search algorithm to generate solutions for the problem is the subject of future research.

Summary

This research shows that munitions inventories can be pre-positioned in a manner that simultaneously improves both objectives of minimizing total costs and minimizing the maximum average coverage value in comparison to the initial solutions. It is also shown that the cost and coverage values achieved by the model depend on the configuration and size of the problem being solved for the six different problems used during the study. In addition, the quality of the solutions in terms of both cost and

coverage is dependent on the combination of transfer size and reset frequency used by the algorithm. The most significant improvements in the quality of solutions for both cost and coverage are evident when using the largest transfer size. Additionally, the most improved solutions are found when the transfer size is combined with the largest reset frequency. The results of the study also provide a means for analyzing which potential warehouse locations should be opened from the set of potential locations and what inventories quantities should possibly be stocked at each location. Finally, the most favorable solutions appear to be found when a small number of new warehouse locations are opened resulting in a reduction of coverage, and a reduction in transportation costs that outweighs the costs of opening the new locations.

The introduction chapter of this dissertation stated the intent of the research to be to provide managers and strategic planners with an improved method for simultaneously making decisions about facility location and inventory positioning problems. The methodology of the study formulates such a combined location and inventory positioning problem and presents a search program for finding solutions to the problem. The results of the study indicate that different search parameters do provide significantly different results for the problem. It is believed that the use of this methodology allows decision makers to understand the tradeoffs inherent in selecting different solutions to the problem and helps them understand the strengths and weaknesses of any location and inventory positioning decision.

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Appendix A

C++ Inventory Program

The following C++ code was used to generate improved solutions to the problem formulated in the Methodology and examined in the experimental design:

```
#include <stdio.h>
#include <stdlib.h>
#include <conio.h>
#include <time.h>
#include <math.h>
#include <iostream.h>

#define n 100
#define t1 25
#define tf 1
#define cooling_rate 95
#define iterations 1000
#define energy1 4
#define pa1 4
#define rad .017453293
#define weapon 10
#define total_locations 100
#define demand_number 50
#define transfer_size 75
#define cover_iteration 450
#define store 20
#define number 100
#define Top_Constant 200000000
#define iterate 40

int h,i,j,k,l,z,z2,x,ii,rr;
long r6,r7;
int huge fill[demand_number+1][number+1];
int m[n+1],con[n+1];
int location[n+1],demand[n+1][n+1],s[n+1],t[n+1],u[n+1],cover_s[n+1];
int potential,requirement,type,s1[n+1];
int surface_requirement,air_requirement;
int temp1,temp2,temp3,temp4,c1,c2,c3,c4,c5,c6,c7,c8,c9,c10;
int c11,c12,c13,c14,c15,c16,c17,c18;
int replace_tracker,cover_open,cover_close;
int quantity[weapon+1][n+1],test_quantity[weapon+1][n+1];
int bestquantity1[weapon+1][n+1];
int cover_quantity[weapon+1][n+1],conflict_quantity[weapon+1][n+1];
int site_demand[n+1];
float dist[n+1][n+1];
float huge best[demand_number+1][number+1];
long double QD,QD_test,QD_best1,QD_best2;
long double QD_best3,QD_best4,QD_best5,QD_best6;
```

```
long double QD_best7,QD_best8,QD_cover;
long double ave,maxave,max_cover;
long double maxave_best1,maxave_best2,maxave_best3,maxave_best4;
long double maxave_best5,maxave_best6,maxave_best7,maxave_best8;
float energy_change,boltzman,temperature;
float pa2,r4,a,b;
float lat[n+1],lon[n+1],surface_cost[n+1],circ[n+1];
float lat1,long1,lat2,long2;
float difflong, difflat;
float bomb_sum,transcost,opencost,closecost;
float short_ton[weapon+1];

time_t timer;
time_t t3,t2;

void initialize(void);
void simulated_annealing(void);
void swap(void);
void decision(void);
void metropolis(void);
void replace(void);
void replace_best(void);
void reset(void);
void map_initial(void);
void distance(void);
void input_data(void);
void input_demand(void);

void main()
{
FILE *fin5,*fin6;
fin5=fopen("out5103.xls","w");
fin6=fopen("qnt5103.xls","w");
for(ii=1;ii<=iterate;ii++)
 {
 initialize();
 cout <<'\n';
 cout <<"Annealing...."<<ii;
 cout <<'\n';
 t3=time(NULL);
 simulated_annealing();
 fprintf(fin5,"%.4Lf",QD_best1);
 fprintf(fin5,"\t%.4Lf",maxave_best1);
 fprintf(fin5,"\t%d",c2);
 fprintf(fin5,"\t%d",c4);
 fprintf(fin5,"\t%.4Lf",QD_best2);
 fprintf(fin5,"\t%.4Lf",maxave_best2);
 fprintf(fin5,"\t%d",c5);
 fprintf(fin5,"\t%d",c6);
 fprintf(fin5,"\t%.4Lf",QD_best3);
 fprintf(fin5,"\t%.4Lf",maxave_best3);
 fprintf(fin5,"\t%d",c7);
 fprintf(fin5,"\t%d",c8);
```

```
 fprintf(fin5,"\t%.4Lf",QD_best4);
 fprintf(fin5,"\t%.4Lf",maxave_best4);
 fprintf(fin5,"\t%d",c9);
 fprintf(fin5,"\t%d",c10);
 fprintf(fin5,"\t%.4Lf",QD_best5);
 fprintf(fin5,"\t%.4Lf",maxave_best5);
 fprintf(fin5,"\t%d",c11);
 fprintf(fin5,"\t%d",c12);
 fprintf(fin5,"\t%.4Lf",QD_best6);
 fprintf(fin5,"\t%.4Lf",maxave_best6);
 fprintf(fin5,"\t%d",c13);
 fprintf(fin5,"\t%d",c14);
 fprintf(fin5,"\t%.4Lf",QD_best7);
 fprintf(fin5,"\t%.4Lf",maxave_best7);
 fprintf(fin5,"\t%d",c15);
 fprintf(fin5,"\t%d",c16);
 fprintf(fin5,"\t%.4Lf",QD_best8);
 fprintf(fin5,"\t%.4Lf",maxave_best8);
 fprintf(fin5,"\t%d",c17);
 fprintf(fin5,"\t%d",c18);
 fprintf(fin5,"\t%.4Lf",QD_cover);
 fprintf(fin5,"\t%.4Lf",max_cover);
 fprintf(fin5,"\t%d",cover_open);
 fprintf(fin5,"\t%d",cover_close);
 fprintf(fin5,"\t%d",t2);
 fprintf(fin5,"\n");
 for(i=1;i<=number;i++)
      {
      fprintf(fin6,"\t%d",cover_s[i]);
      for(j=1;j<=weapon;j++)
       {
       fprintf(fin6,"\t%d",cover_quantity[j][cover_s[i]]);
       }
      fprintf(fin6,"\n");
      }
 }
fclose(fin5);
fclose(fin6);
cout <<'\n';
cout <<"Done ...";
}

void initialize(void)
{
//set the random seed and annealing parameters//
long double zz;
float air;
srand((unsigned) time(&timer));
boltzman=((float)-
energy1/(float)100)/((float)t1*log((float)pa1/(float)100));
temperature=t1;
r6=0; //tracks number of transfers//
r7=0; //tracks number of coverage resets//
```

```
c2=0;c4=0;c5=0;c6=0;c7=0;c8=0;c9=0;c10=0;c11=0;
c12=0;c13=0;c14=0;c15=0;c16=0;c17=0;c18=0;
//Set the number of different munitions types and their tonnages //
rr=0;
type=10;
short_ton[1]=1.055125;
short_ton[2]=.303335;
short_ton[3]=1.186375;
short_ton[4]=1.0925;
short_ton[5]=.30155;
short_ton[6]=1.05361;
short_ton[7]=.3975;
short_ton[8]=.249735;
short_ton[9]=1.1125;
short_ton[10]=.523;

//Input the Lat/Longitudes, routing, trans. costs by location//

input_data();

//Set the locations parameters for the problem//

potential=total_locations; //all locations are potential sites - This
could be restricted//
for (i=1;i<=potential;i++)
 {
  t[i]=i;
 }

//Reset and read-in Initial Site locations and stock quantities//

for (i=1;i<=total_locations;i++)
{
 for (j=1;j<=type;j++)
  {
  quantity[j][i]=0;
  }
}

FILE *fin1;
fin1=fopen("initial.txt","r");
for (i=1;i<=store;i++)
 {
  fscanf(fin1,"%d",&s[i]);
  u[i]=s[i];
  for (j=1;j<=type;j++)
  {
       fscanf(fin1,"%d",&quantity[j][s[i]]);
  }
 }
fclose(fin1);

//Determine the additional sites initially considered//
```

```
j=number-store;
 for (i=1;i<=j;i++)
  {
  s[store+i]= t[random(potential)+1];
  for (k=1;k<=(store+i-1);k++)
      {
      if (s[store+i]==s[k]) i=i-1;
      }
  }

//Haversine Distance Calculations using Lat and Long//

for (i=1;i<=total_locations;i++)
  {
  for (j=1;j<=total_locations;j++)
      {
      lat1=lat[i];
      lat2=lat[j];
      long1=lon[i];
      long2=lon[j];

      //Convert Decimal Degrees to Radians for Trig Calculations//
      lat1=lat1*rad;
      lat2=lat2*rad;
      long1=long1*rad;
      long2=long2*rad;

      //Calculate Distance in miles//
      difflat=(lat2-lat1);
      difflong=(long2-long1);
      a=(sin(difflat/2)*sin(difflat/2))
      +(cos(lat1)*cos(lat2)*sin(difflong/2)*sin(difflong/2));
      b=2*atan2(sqrt(a),sqrt(1-a));
      dist[i][j]=b*3956;
      }
  }
cout <<'\n';
cout<<"The Demand Size is "<<demand_number;
cout<<'\n';
cout<<"The number of total locations is "<<total_locations;
cout<<'\n';
cout<<"The transfer size is "<<transfer_size;
cout<<'\n';
cout<<"The cover reset size is "<<cover_iteration;
cout<<'\n';

//Determine Combat Locations and demand//

input_demand();

for(i=1;i<=demand_number;i++)
 {
      site_demand[i]=0;
```

```
      for(j=1;j<=type;j++)
      {
      site_demand[i]=site_demand[i]+demand[j][i];
      }
 }

//Compute Initial Quantity Distances and closest supply locations//
distance();

//Compute Initial Objective Function Value//
 ave=0;
 QD=0;
 max_cover=0;
 for(i=1;i<=total_locations;i++)
 {
  for(j=1;j<=type;j++)
      {
      conflict_quantity[j][i]=quantity[j][i];
      }
 }

 for (i=1;i<=demand_number;i++)
 {
  if(i>1)
  {
  if(con[i]!=con[i-1])
      {
       for(l=1;l<=total_locations;l++)
       {
        for(j=1;j<=type;j++)
        {
        conflict_quantity[j][l]=quantity[j][l];
        }
       }
      }
  }
  zz=0;
  for (k=1;k<=type;k++)
  {
  requirement=demand[k][i];
  j=1;
  while (requirement>0)
      {
      if(best[i][j]<=500){air=4.0;}
      else
             {
             if(best[i][j]<=1000){air=2.0;}
             else
                    {
                    if(best[i][j]<=4000){air=1.0;}
                    else
                           {air=.75;}
                    }
```

```
            }
      if (conflict_quantity[k][fill[i][j]]>=requirement)
       {
       air_requirement=.10*requirement;
       surface_requirement=requirement-air_requirement;
       QD=QD+((long double)surface_requirement*(long
       double)short_ton[k]*(long double)best[i][j]*(long
       double)surface_cost[fill[i][j]]*(long double)circ[fill[i][j]]);
       zz=zz+(long double)surface_requirement*(long
       double)best[i][j]*(long double)circ[fill[i][j]];
       QD=QD+(long double)air_requirement*(long double)best[i][j]*(long
       double)air*(long double)short_ton[k];
       zz=zz+(long double)air_requirement*(long double)best[i][j];
       requirement=0;
      conflict_quantity[k][fill[i][j]]=conflict_quantity[k][fill[i][j]]
      -requirement;
       }
      else
       {
       air_requirement=.10*conflict_quantity[k][fill[i][j]];
       surface_requirement=conflict_quantity[k][fill[i][j]]-
       air_requirement;
       QD=QD+((long double)surface_requirement*(long double)
       short_ton[k]*(long double)best[i][j]*(long
       double)surface_cost[fill[i][j]]*(long double)circ[fill[i][j]]);
       zz=zz+(long double)surface_requirement*(long
       double)best[i][j]*(long double)circ[fill[i][j]];
       QD=QD+(long double)air_requirement*(long double)best[i][j]*(long
       double)air*(long double)short_ton[k];
       zz=zz+(long double)air_requirement*(long double)best[i][j];
       requirement=requirement-conflict_quantity[k][fill[i][j]];
       conflict_quantity[k][fill[i][j]]=0;
       j=j+1;
       }
      }
  }
 ave=zz/(long double)site_demand[i];
 if (ave>max_cover)
  {
  max_cover=ave;
  }
 }
cout <<'\n';
cout <<"Initial Maximum Average Coverage Distance = ";
cout <<max_cover;
maxave_best1=max_cover;
maxave_best2=max_cover;
maxave_best3=max_cover;
maxave_best4=max_cover;
maxave_best5=max_cover;
maxave_best6=max_cover;
maxave_best7=max_cover;
maxave_best8=max_cover;
```

```
cout <<'\n';
cout <<"The total Quantity-Distance after meeting
conflict"<<demand_number<<
" is = "<<QD<<'\n';
QD_best8=Top_Constant;
QD_best7=Top_Constant;
QD_best6=Top_Constant;
QD_best5=Top_Constant;
QD_best4=Top_Constant;
QD_best3=Top_Constant;
QD_best2=Top_Constant;
QD_best1=Top_Constant;
QD_cover=QD;

//Initialize coverage variables//
for (i=1;i<=total_locations;i++)
 {
 for (k=1;k<=type;k++)
  {cover_quantity[k][i]=quantity[k][i];}
 }
for (i=1;i<=number;i++)
 {cover_s[i]=s[i];}
}
void distance(void)
{
/*Initialize Best Distances and locations*/
for (i=1;i<=demand_number;i++)
 {
 for (j=1;j<=number;j++)
  {
  best[i][j]=25999; //value larger than any distance on earth//
  fill[i][j]=0;
  }
 }

//Determine Closest Supply Locations and Distances//

for(i=1;i<=demand_number;i++)
 {
 for(j=1;j<=number;j++)
  {
      x=0;
      for(k=1;k<=number;k++)
      {
       if(x==0)
       {
        if (dist[s[j]][location[i]]<best[i][k])
        {
            for(l=number;l>=k+1;l--)
             {
              fill[i][l]=fill[i][l-1];
              best[i][l]=best[i][l-1];
             }
```

```
            fill[i][k]=s[j];
            best[i][k]=dist[s[j]][location[i]];
            x=1;
        }
       }
      }
  }}}

void simulated_annealing(void)
{
while(temperature>tf)
 {
 for(h=1;h<=iterations;h++)
  {
  swap();
  }
 temperature=temperature*(cooling_rate/(float)100);
 }
}

void swap(void)
{
 int r1,r2,r5,ware;
 long double xx;
 float air2;
 temp1=0,temp2=0;
 temp3=0,temp4=0;
 r6=r6+1;
 rr=rr+1;

 //set test values//

 for(i=1;i<=total_locations;i++)
      {
      for(k=1;k<=type;k++)
       {
       test_quantity[k][i]=quantity[k][i];
       }
      }

 //Decide on type of transfer to make: location or inventory//
 pa2=.05;
 r4=rand()/(float)32767;
 if (r4<pa2)
 {
  //Identify locations to switch//
  //Ensure new location is not a storage location already//
  j=0;
  temp3=random(number)+1;
  bomb_sum=0;
  for(i=1;i<=type;i++)
      {
      bomb_sum=bomb_sum+(test_quantity[i][s[temp3]]*short_ton[i]);
```

```
      }
 while (j==0)
 {
      j=1;
      temp4=random(potential)+1;
      for(i=1;i<=number;i++)
      {
       if(s[i]==t[temp4]) j=0;
      }
      if(bomb_sum>m[t[temp4]]) j=0;
 }
//Make Transfer//
 for(k=1;k<=type;k++)
      {
      test_quantity[k][t[temp4]]=test_quantity[k][s[temp3]];
      test_quantity[k][s[temp3]]=0;
      }
 z2=s[temp3];
 s[temp3]=t[temp4];
 distance();
}
else
{
//find targets locations with inventory quantities to transfer//
while(temp1==0)
 {
 r1=random(number)+1;
 bomb_sum=0;
 for(i=1;i<=type;i++)
      {
      bomb_sum=bomb_sum+(test_quantity[i][s[r1]]*short_ton[i]);
      }
 if (bomb_sum<m[s[r1]])
      {
      temp1=r1;
      }
 else
 temp1=0;
 }
r5=random(type)+1; //Equal chance of a transfer for each type weapon//
while (temp2==0)
 {
 r2=random(number)+1;
 if(r2!=temp1)
      {
      if(test_quantity[r5][s[r2]]>0)
            {
            temp2=r2;
            if(test_quantity[r5][s[r2]]>=transfer_size)
             {
             //Make Transfer//

             test_quantity[r5][s[temp1]]=test_quantity[r5][s[temp1]]
```

```
            +transfer_size;
            test_quantity[r5][s[temp2]]=test_quantity[r5][s[temp2]]-
            transfer_size;
            }
           else
            {
            test_quantity[r5][s[temp1]]=test_quantity[r5][s[temp1]]
            +test_quantity[r5][s[temp2]];
            test_quantity[r5][s[temp2]]=0;
            }
           }
      else
      temp2=0;
      }
  else
  temp2=0;
  }
 }

//Compute QD of test solution//
QD_test=0;
maxave=0;
for(i=1;i<=total_locations;i++)
 {
 for(j=1;j<=type;j++)
  {
  conflict_quantity[j][i]=test_quantity[j][i];
  }
 }
for(i=1;i<=demand_number;i++)
 {
 if(i>1)
  {
  if(con[i]!=con[i-1])
      {
       for(l=1;l<=total_locations;l++)
       {
        for(j=1;j<=type;j++)
            {
            conflict_quantity[j][l]=test_quantity[j][l];
            }
       }}}
 xx=0;
 for (k=1;k<=type;k++)
  {
  requirement=demand[k][i];
  j=1;
  while (requirement>0)
      {
      if(best[i][j]<=500){air2=4.0;}
      else
            {
            if(best[i][j]<=1000){air2=2.0;}
```

```
            else
                  {
                  if(best[i][j]<=4000){air2=1.0;}
                  else
                        {air2=.75;}
                  }
             }
      if (conflict_quantity[k][fill[i][j]]>=requirement)
       {
       air_requirement=.10*requirement;
       surface_requirement=requirement-air_requirement;
       QD_test=QD_test+((long double)surface_requirement*(long
       double)short_ton[k]*(long double)best[i][j]*(long
       double)surface_cost[fill[i][j]]*(long double)circ[fill[i][j]]);
       xx=xx+(long double)surface_requirement*(long
       double)best[i][j]*(long double)circ[fill[i][j]];
       QD_test=QD_test+(long double)air_requirement*(long
       double)best[i][j]*(long double)air2*(long double)short_ton[k];
       xx=xx+(long double)air_requirement*(long double)best[i][j];
       requirement=0;
       conflict_quantity[k][fill[i][j]]=
       conflict_quantity[k][fill[i][j]]-requirement;
       }
      else
       {
       air_requirement=.10*conflict_quantity[k][fill[i][j]];
       surface_requirement=conflict_quantity[k][fill[i][j]]-
       air_requirement;
       QD_test=QD_test+((long double)surface_requirement*(long
       double)short_ton[k]*(long double)best[i][j]*(long
       double)surface_cost[fill[i][j]]*(long double)circ[fill[i][j]]);
       xx=xx+(long double)surface_requirement*(long
       double)best[i][j]*(long double)circ[fill[i][j]];
       QD_test=QD_test+(long double)air_requirement*(long
       double)best[i][j]*(long double)air2*(long double)short_ton[k];
       xx=xx+(long double)air_requirement*(long double)best[i][j];
       requirement=requirement-conflict_quantity[k][fill[i][j]];
       conflict_quantity[k][fill[i][j]]=0;
       j=j+1;
       }
      }
 }
ave=xx/(long double)site_demand[i];
if(ave>maxave) maxave=ave;
}
transcost=QD_test;

 //Add opening and closing costs for sites//

c1=0;
c3=0;
for (i=1;i<=number;i++)
{
```

```
 bomb_sum=0;
 k=0;
 for (j=1;j<=store;j++)
      {
      if (s[i]==u[j])
      {k=1;}
      }
 if (k==0)
      {
      for (k=1;k<=type;k++)
       {
       bomb_sum=bomb_sum+test_quantity[k][s[i]]*short_ton[k];
       }
       if (bomb_sum>0)
       {
       ware=(bomb_sum/250)+1;
       QD_test=QD_test+(ware*1000000)+2000000;
       c1=c1+1;
       }
      }
}
opencost=QD_test;
for (i=1;i<=store;i++)
{
 bomb_sum=0;
 for (k=1;k<=type;k++)
      {
      bomb_sum=bomb_sum+test_quantity[k][u[i]];
      }
 if (bomb_sum==0)
      {
      QD_test=QD_test+200000;    // Closing cost of sites based on
                                    original size and cost//
      c3=c3+1;
      }
}
closecost=QD_test;
decision();
if(replace_tracker>0)
 {
 if(maxave<max_cover)
      {
  t2=time(NULL)-t3;
      r7=r7+1;
      QD_cover=QD_test;
      max_cover=maxave;
      cover_open=c1;
      cover_close=c3;
      for(i=1;i<=total_locations;i++)
       {
       for(k=1;k<=type;k++)
        {cover_quantity[k][i]=test_quantity[k][i];}
       }
```

```
      for(i=1;i<=number;i++)
       {cover_s[i]=s[i];}
      }
  }
 if(rr==cover_iteration)
  {
  rr=0;
  reset();
  }
}

void decision(void)
{
if(QD_test<QD){replace();}
else metropolis();
}

void metropolis(void)
{
float pa,r3;
energy_change=(QD_test-QD)/QD;
pa=exp((-energy_change/(boltzman*temperature)));
r3=rand()/(float)32767;
if(r3<pa)
 {
 replace();
 }
else
 {
 replace_tracker=0;
 if(r4<pa2)
  {
  s[temp3]=z2;
  distance();
  }
 }
}
void replace(void)
{
replace_tracker=1;
for(i=1;i<=total_locations;i++)
 {
 for(k=1;k<=type;k++)
  {
  quantity[k][i]=test_quantity[k][i];
  }
 }
QD=QD_test;
if(maxave<=3000)
 {
 if(QD_test<QD_best1)replace_best();
 }
else
```

```
 {
 if(maxave<=3250)
  {
  if(QD_test<QD_best2)replace_best();
  }
 else
      {
      if(maxave<=3500)
       {
       if(QD_test<QD_best3)replace_best();
       }
      else
       {
        if(maxave<=3750)
            {
            if(QD_test<QD_best4)replace_best();
            }
        else
            {
            if(maxave<=4000)
             {
             if(QD_test<QD_best5)replace_best();
             }
            else
             {
             if(maxave<=4250)
              {
              if(QD_test<=QD_best6)replace_best();
              }
             else
              {
              if(maxave<=4500)
                   {
                   if(QD_test<=QD_best7)replace_best();
                   }
              else
                   {
                   if(QD_test<=QD_best8)replace_best();
                   }
              }}}}}}}

void replace_best(void)
{
if(maxave<=3000)
 {
 maxave_best1=maxave;
 QD_best1=QD_test;
 c2=c1;
 c4=c3;
 for(i=1;i<=total_locations;i++)
  {
  for(k=1;k<=type;k++)
       {
```

```
        bestquantity1[k][i]=test_quantity[k][i];
        }
   }
  for(i=1;i<=number;i++)
   {s1[i]=s[i];}
  }
 else
  {
   if(maxave<=3250)
        {
         maxave_best2=maxave;
         QD_best2=QD_test;
         c5=c1;
         c6=c3;
         }
   else
   {
        if(maxave<=3500)
        {
        maxave_best3=maxave;
        QD_best3=QD_test;
        c7=c1;
        c8=c3;
        }
        else
        {
         if(maxave<=3750)
         {
         maxave_best4=maxave;
         QD_best4=QD_test;
         c9=c1;
         c10=c3;
         }
         else
         {
          if(maxave<=4000)
          {
          maxave_best5=maxave;
          QD_best5=QD_test;
          c11=c1;
          c12=c3;
          }
          else
          {
              if(maxave<=4250)
               {
               maxave_best6=maxave;
               QD_best6=QD_test;
               c13=c1;
               c14=c3;
               }
              else
               {
```

```
              if(maxave<=4500)
               {
               maxave_best7=maxave;
               QD_best7=QD_test;
               c15=c1;
               c16=c3;
               }
              else
               {
               maxave_best8=maxave;
               QD_best8=QD_test;
               c17=c1;
               c18=c3;
               }
              }
         }}}}}}

void reset(void)
{
 QD=QD_cover;
 for(i=1;i<=total_locations;i++)
  {
      for(k=1;k<=type;k++)
       {
       quantity[k][i]=cover_quantity[k][i];
       }
  }
 k=0;
 for(i=1;i<=number;i++) //check if the location set is different//
  {
      for(j=1;j<=number;j++)
       {
        if (cover_s[i]==s[j])
            {k=k+1;}
       }
  }
 for(i=1;i<=number;i++)
  {s[i]=cover_s[i];}

 if(k<number)
  {
  distance();
  }
}

void map_initial(void)
{
cout <<"Here's the initial locations and quantities"<<'\n';
for(i=1;i<=number;i++)
 {
 cout <<'\n';
 cout <<"At location ";
 cout << s[i];
```

```
 cout <<'\n';
 for (j=1;j<=type;j++)
  {
  cout <<".......";
  cout << quantity[j][s[i]];
  cout <<" of munitions type "<<j;
  cout <<'\n';
  }
 }
}

void input_data ()
{
FILE *fin1,*fin2,*fin3,*fin4,*fin5;
fin1=fopen("latitude.txt","r");
fin2=fopen("longitud.txt","r");
fin3=fopen("circuit.txt","r");
fin4=fopen("surface.txt","r");
fin5=fopen("Capacity.txt","r");

for (i=1;i<=total_locations;i++)
 {
  fscanf(fin1,"%f",&lat[i]);
  fscanf(fin2,"%f",&lon[i]);
  fscanf(fin3,"%f",&circ[i]);
  fscanf(fin4,"%f",&surface_cost[i]);
  fscanf(fin5,"%f",&m[i]);
 }
fclose(fin1);
fclose(fin2);
fclose(fin3);
fclose(fin4);
fclose(fin5);
}

void input_demand ()
{
FILE *fin1,*fin2;
fin1=fopen("Mix.txt","r");
fin2=fopen("Mixcon.txt","r");
for(i=1;i<=demand_number;i++)
 {
 fscanf(fin1,"%d",&location[i]);
 fscanf(fin2,"%d",&con[i]);
 for(j=1;j<=type;j++)
  {
  fscanf(fin1,"%d",&demand[j][i]);
  }
 }
 fclose(fin1);
 fclose(fin2);
}
```

Appendix B

Location and Inventory Data

The following data is used to represent the locations, weapons, inventory supply and inventory demand used in the C++ program for the study. The locations and their descriptive variable values are listed in the Figure B1.

#	Location	Circ	Surface$	Lat.	Long.	Surface Costs	$
1	Lakenheath, UK	1.3	0.12	52.4	0.55	Pacific	0.3
2	Alconbury, UK	1.3	0.12	52.383	-0.2167	Asia	0.18
3	Bentwaters, UK	1.3	0.12	52.133	1.43333	Europe	0.12
4	Rota, SP	1.3	0.12	36.633	6.36667		
5	Fairford, UK	1.3	0.12	51.683	-1.7833	Air Rate	$/mile/ton
6	Sigonella, IT	1.3	0.12	37.5	15.0833	<500	4
7	Volkel, NE	1.3	0.12	51.65	5.7	<1500	2
8	Soesterburg, NE	1.3	0.12	52.117	5.31667	<4000	1
9	Sollingen, GE	1.3	0.12	48.783	8.08333	>4000	0.75
10	Sembach, GE	1.3	0.12	49.5	7.86667		
11	Ramstein, GE	1.3	0.12	49.433	7.6		
12	Rhein Mein, GE	1.3	0.12	50.017	8.56667	Circ. Routing	
13	Spangdahlem, GE	1.3	0.12	49.983	6.7		
14	Weisbaden, GE	1.3	0.12	50.05	8.33333	Europe	1.3
15	Kunchyna, SLO	1.3	0.12	48.4	17.15	Asia	1.5
16	Norvenich, GE	1.3	0.12	50.833	6.66667	Pacific	1.7
17	Hahn, GE	1.3	0.12	49.933	7.25	No Air	+ 0.2
18	Minsk, BEL	1.3	0.12	53.633	27.4833		
19	Bremgarten, GE	1.3	0.12	47.9	7.61667		
20	Ahlhorn, GE	1.3	0.12	52.883	8.23333		
21	Tazlar, HUN	1.3	0.12	46.55	19.5		
22	Tuzla, BOS	1.3	0.12	44.533	18.6667		
23	Slupsk, POL	1.3	0.12	54.467	17.0333		
24	Zaragoza, SP	1.3	0.12	41.633	-0.8833		
25	Torrejon, SP	1.3	0.12	40.483	-3.4667		
26	Moron, SP	1.3	0.12	37.167	-5.6		
27	Aviano, IT	1.3	0.12	46.017	12.7		
28	Camp Darby, IT	1.5	0.12	43.65	10.35		
29	Livorno, IT	1.3	0.12	43.55	10.3167		
30	Ghedi, IT	1.3	0.12	45.433	10.2667		
31	Larisa, GR	1.3	0.12	39.65	22.45		
32	Nea Anchialos, GR	1.3	0.12	39.217	22.8		
33	Vandel, NO	1.3	0.12	55.683	9.18333		

34	Bodo, NO	1.3	0.12	67.267	14.3667
35	Poznan, PO	1.3	0.12	52.4	16.9
36	Krzesiny, PO	1.3	0.12	52.35	16.9833
37	Andoya, NO	1.3	0.12	69.283	16.1333
38	Incirlik, TU	1.3	0.12	37	35.4333
39	Ankara, TU	1.3	0.12	40.083	32.5667
40	Murted, TU	1.3	0.12	40.067	32.55
41	Balikeshir, TU	1.3	0.12	39.617	27.9333
42	Eskeshir, TU	1.3	0.12	39.783	30.5833
43	Izmir, TU	1.3	0.12	38.317	27.1667
44	Iskendrun, TU	1.5	0.12	36.5	35.6667
45	Batman, TU	1.3	0.12	37.917	41.1167
46	Diyarbakir, TU	1.3	0.12	37.9	40.2
47	Constanta, RO	1.3	0.12	44.183	28.65
48	Prince Sultan, SA	1.5	0.18	24.067	47.5833
49	Dhahran, SA	1.5	0.18	26.267	50.15
50	King Khalid, SA	1.5	0.18	27.883	45.5167
51	Al Jubayl	1.5	0.18	27	49.4167
52	Muscat	1.5	0.18	23.6	58.55
53	Dubai	1.5	0.18	25.25	55.2667
54	Masirah	1.5	0.18	20.417	58.7333
55	Thumrait	1.5	0.18	17.667	54.0333
56	Shaikh Isa BA	1.5	0.18	25.917	50.5833
57	Al Dhafra	1.5	0.18	24.25	54.55
58	Abu Dhabi, UAE	1.5	0.18	24.467	54.3667
59	Darwin, AUS	1.7	0.3	12.467	130.833
60	Kadena, JA	1.7	0.3	26.35	127.767
61	Anderson Guam	1.7	0.3	13.567	144.917
62	Kunsan, KO	1.7	0.3	35.983	126.717
63	Misawa, JA	1.7	0.3	40.7	141.35
64	Yokota, JA	1.7	0.3	35.733	139.333
65	Osan, KO	1.7	0.3	37.133	127.067
66	Kwonju, KO	1.7	0.3	35.117	126.8
67	Sachon, KO	1.7	0.3	35.083	128.083
68	Taegu, KO	1.7	0.3	35.867	128.583
69	Suwon, KO	1.7	0.3	37.233	127
70	Diego Garcia, UK	1.7	0.3	-7.25	72.4
71	Eielson Alaska	1.7	0.3	64.65	147.083
72	Elmendorf Alaska	1.7	0.3	61.25	149.783
73	Naha, JA	1.9	0.3	26.2	127.667
74	Tengen, JA	1.7	0.3	26.367	127.85
75	Chen Hae, KO	1.7	0.3	35.15	128.7
76	Taipei, TAI	1.7	0.3	25.033	121.517
77	Bangkok, THAI	1.7	0.3	13.75	100.517
78	Al Jaber KU	1.5	0.18	29.35	47.6833
79	Baguio, PHIL	1.7	0.3	16.367	120.617
80	Clark, PHIL	1.7	0.3	14.6	120.983

81	Paya Lebar, SING	1.7	0.3	1.3667	103.917
82	Nakhan Phan, THAI	1.7	0.3	17.4	104.767
83	Korat, THAI	1.7	0.3	14.6	102.083
84	Dowidz, POL	1.3	0.12	51.433	21.2833
85	Heart, AFG	1.5	0.18	34.35	62.1833
86	Kandahar, AFG	1.5	0.18	31.5	65.85
87	Bagram, AFG	1.5	0.18	34.967	69.2833
88	Erzurum, TU	1.3	0.12	39.9	41.2833
89	Bukhoro, UZBEK	1.5	0.18	39.767	64.4333
90	Ashgabat, TURKM	1.5	0.18	37.95	58.3833
91	Dushhanb, TAJ	1.5	0.18	38.567	68.7667
92	Antalya, TU	1.3	0.12	36.883	30.7167
93	Islamabad, PAK	1.5	0.18	33.617	73.1
94	Tashkent, UZBEK	1.5	0.18	41.3	69.2833
95	Bishkek, KYRG	1.5	0.18	42.85	74.5667
96	Chagai, PAK	1.5	0.18	29.3	64.7
97	Quetta, PAK	1.5	0.18	30.2	67.0167
98	Baku, AZER	1.5	0.18	40.367	49.8167
99	Haifa, IS	1.5	0.18	32.8	35
100	Tel Aviv IS	1.5	0.18	32.067	34.7833

Figure B1. Location Data

The initial inventory quantities are provided in Figure B2.

Initial Munitions Positioning

	Weapon #									
Loc	1	2	3	4	5	6	7	8	9	10
1	62	314	191	35	3112	323	50	100	907	20
5	124	310	63	0	4164	464	0	0	1718	61
26	62	288	63	0	2082	232	0	0	1079	61
11	348	321	412	82	11328	1572	376	592	2850	154
13	62	181	300	35	3113	323	166	428	907	45
27	62	182	300	35	3113	323	50	100	908	45
28	186	166	310	62	6246	695	125	397	1618	100
38	247	208	371	162	8480	703	156	696	900	183
48	529	350	731	190	11206	1308	376	934	2642	305
49	393	129	433	32	2241	262	175	187	1728	61
51	318	178	399	97	6724	785	226	439	2185	183
50	365	350	700	190	11206	785	505	493	3041	305
70	347	42	0	42	8906	1554	0	1120	3353	236

61	399	254	205	49	13088	1468	107	288	2908	332
60	332	212	716	170	10906	1224	376	1007	2840	277
63	165	113	300	50	4000	400	150	400	900	120
62	115	100	300	50	5000	600	150	400	1000	120
65	120	100	299	100	5000	600	150	400	1000	120
55	318	178	399	97	6724	785	226	561	2185	183
72	132	27	225	69	3450	358	141	383	2044	79
Total	4686	4003	6717	1547	130089	14764	3505	8925	36713	2990

Figure B2. Initial Inventory Position

The Weapon weights are provided in Figure B3.

Weapon	Pounds	Tons
1	2110.25	1.055125
2	606.67	0.303335
3	2372.75	1.186375
4	2185	1.0925
5	603.1	0.30155
6	2107.22	1.05361
7	795	0.3975
8	499.47	0.249735
9	2225	1.1125
10	1046	0.523

Figure B3. Weapon Weights

The demand set for the Major Theater War scenario is provided in Figure B4.

Dem #	Con #	Loc #	Weapon # 1	2	3	4	5	6	7	8	9	10
1	1	49	150	1000	500	0	3000	100	2200	500	250	500
2	1	38	100	200	0	100	500	0	0	0	0	100
3	1	78	100	500	150	0	3500	1800	0	2500	75	250
4	1	50	0	0	0	0	100	1500	0	1000	0	0
5	1	48	500	2000	750	500	1500	350	800	150	375	1000
6	2	62	300	1000	1000	0	6000	200	2200	1000	500	500
7	2	60	200	200	0	200	1000	0	0	0	0	100
8	2	61	200	500	300	0	7000	3600	0	5000	150	250
9	2	63	0	0	0	0	200	3000	0	2000	0	0
10	2	65	1000	2000	1500	1000	3000	700	800	300	750	1000
11	3	55	150	1000	500	0	3000	100	2200	500	250	500
12	3	38	100	200	0	100	500	0	0	0	0	100
13	3	78	100	500	150	0	3500	1800	0	2500	75	250
14	3	50	0	0	0	0	100	1500	0	1000	0	0
15	3	48	500	2000	750	500	1500	350	800	150	375	1000
16	4	62	300	1000	1000	0	6000	200	2200	1000	500	500
17	4	60	200	200	0	200	1000	0	0	0	0	100
18	4	61	200	500	300	0	7000	3600	0	5000	150	250
19	4	63	0	0	0	0	200	3000	0	2000	0	0
20	4	64	1000	2000	1500	1000	3000	700	800	300	750	1000
21	5	56	150	1000	500	0	3000	100	2200	500	250	500
22	5	38	100	200	0	100	500	0	0	0	0	100
23	5	78	100	500	150	0	3500	1800	0	2500	75	250
24	5	50	0	0	0	0	100	1500	0	1000	0	0
25	5	48	500	2000	750	500	1500	350	800	150	375	1000
26	6	62	300	1000	1000	0	6000	200	2200	1000	500	500
27	6	60	200	200	0	200	1000	0	0	0	0	100
28	6	61	200	500	300	0	7000	3600	0	5000	150	250
29	6	63	0	0	0	0	200	3000	0	2000	0	0
30	6	70	1000	2000	1500	1000	3000	700	800	300	750	1000
31	7	61	500	300	300	300	300	300	300	300	150	150
32	7	60	500	200	200	200	200	200	200	200	100	100
33	7	62	300	150	150	150	150	150	150	150	75	75
34	8	1	100	150	150	150	150	150	150	150	75	75
35	8	11	250	300	300	300	300	300	300	300	150	150
36	8	38	250	200	200	200	200	200	200	200	100	100
37	8	27	200	250	250	250	250	250	250	250	125	125
38	9	48	500	300	300	300	300	300	300	300	150	150
39	9	50	500	200	200	200	200	200	200	200	100	100
40	9	49	300	150	150	150	150	150	150	150	75	75
41	10	81	300	150	150	150	150	150	150	150	75	75

42	11	36	200	250	250	250	250	250	250	250	125	125
43	12	93	300	150	150	150	150	150	150	150	75	75
44	13	89	300	150	150	150	150	150	150	150	75	75
45	14	33	200	250	250	250	250	250	250	250	125	125
46	15	80	300	150	150	150	150	150	150	150	75	75
47	16	74	300	150	150	150	150	150	150	150	75	75
48	17	15	200	250	250	250	250	250	250	250	125	125
49	18	83	300	150	150	150	150	150	150	150	75	75
50	19	30	200	250	250	250	250	250	250	250	125	125
51	20	49	150	1000	500	0	3000	100	2200	500	250	500
52	20	38	100	200	0	100	500	0	0	0	0	100
53	20	78	100	500	150	0	3500	1800	0	2500	75	250
54	20	50	0	0	0	0	100	1500	0	1000	0	0
55	20	48	500	2000	750	500	1500	350	800	150	375	1000
56	21	62	300	1000	1000	0	6000	200	2200	1000	500	500
57	21	60	200	200	0	200	1000	0	0	0	0	100
58	21	61	200	500	300	0	7000	3600	0	5000	150	250
59	21	63	0	0	0	0	200	3000	0	2000	0	0
60	21	65	1000	2000	1500	1000	3000	700	800	300	750	1000
61	22	53	150	1000	500	0	3000	100	2200	500	250	500
62	22	38	100	200	0	100	500	0	0	0	0	100
63	22	78	100	500	150	0	3500	1800	0	2500	75	250
64	22	50	0	0	0	0	100	1500	0	1000	0	0
65	22	48	500	2000	750	500	1500	350	800	150	375	1000
66	23	62	300	1000	1000	0	6000	200	2200	1000	500	500
67	23	60	200	200	0	200	1000	0	0	0	0	100
68	23	61	200	500	300	0	7000	3600	0	5000	150	250
69	23	63	0	0	0	0	200	3000	0	2000	0	0
70	23	71	1000	2000	1500	1000	3000	700	800	300	750	1000
71	24	51	150	1000	500	0	3000	100	2200	500	250	500
72	24	38	100	200	0	100	500	0	0	0	0	100
73	24	78	100	500	150	0	3500	1800	0	2500	75	250
74	24	50	0	0	0	0	100	1500	0	1000	0	0
75	24	48	500	2000	750	500	1500	350	800	150	375	1000
76	25	62	300	1000	1000	0	6000	200	2200	1000	500	500
77	25	60	200	200	0	200	1000	0	0	0	0	100
78	25	61	200	500	300	0	7000	3600	0	5000	150	250
79	25	63	0	0	0	0	200	3000	0	2000	0	0
80	25	68	1000	2000	1500	1000	3000	700	800	300	750	1000
81	26	61	500	300	300	300	300	300	300	300	150	150
82	26	60	500	200	200	200	200	200	200	200	100	100
83	26	62	300	150	150	150	150	150	150	150	75	75
84	27	1	100	150	150	150	150	150	150	150	75	75
85	27	11	250	300	300	300	300	300	300	300	150	150
86	27	38	250	200	200	200	200	200	200	200	100	100
87	27	27	200	250	250	250	250	250	250	250	125	125
88	28	48	500	300	300	300	300	300	300	300	150	150

89	28	50	500	200	200	200	200	200	200	200	100	100
90	28	49	300	150	150	150	150	150	150	150	75	75
91	29	72	300	150	150	150	150	150	150	150	75	75
92	30	75	300	150	150	150	150	150	150	150	75	75
93	31	90	300	150	150	150	150	150	150	150	75	75
94	32	73	300	150	150	150	150	150	150	150	75	75
95	33	28	200	250	250	250	250	250	250	250	125	125
96	34	47	200	250	250	250	250	250	250	250	125	125
97	35	58	300	150	150	150	150	150	150	150	75	75
98	36	59	300	150	150	150	150	150	150	150	75	75
99	37	77	300	150	150	150	150	150	150	150	75	75
100	38	3	200	250	250	250	250	250	250	250	125	125

Figure B4. MTW Demand Set

The demand set for the Small Scale Contingency (SSC) scenario is listed in Figure B5.

Dem	Con	Loc				Weapon #						
#	#	#	1	2	3	4	5	6	7	8	9	10
1	1	49	150	1000	500	0	3000	100	2200	500	250	500
2	1	38	100	200	0	100	500	0	0	0	0	100
3	1	78	100	500	150	0	3500	1800	0	2500	75	250
4	1	50	0	0	0	0	100	1500	0	1000	0	0
5	1	48	500	2000	750	500	1500	350	800	150	375	1000
6	2	62	300	1000	1000	0	6000	200	2200	1000	500	500
7	2	60	200	200	0	200	1000	0	0	0	0	100
8	2	61	200	500	300	0	7000	3600	0	5000	150	250
9	2	63	0	0	0	0	200	3000	0	2000	0	0
10	2	65	1000	2000	1500	1000	3000	700	800	300	750	1000
11	3	61	500	300	300	300	300	300	300	300	300	300
12	3	60	500	200	200	200	200	200	200	200	200	200
13	3	62	300	150	150	150	150	150	150	150	150	150
14	4	1	100	150	150	150	150	150	150	150	150	150
15	4	11	250	300	300	300	300	300	300	300	300	300
16	4	38	250	200	200	200	200	200	200	200	200	200
17	4	27	200	250	250	250	250	250	250	250	250	250
18	5	48	500	300	300	300	300	300	300	300	300	300
19	5	50	500	200	200	200	200	200	200	200	200	200
20	5	49	300	150	150	150	150	150	150	150	150	150
21	6	70	500	300	300	300	300	300	300	300	300	300
22	6	60	500	200	200	200	200	200	200	200	200	200

23	6	62	300	150	150	150	150	150	150	150	150	150
24	7	99	100	150	150	150	150	150	150	150	150	150
25	7	11	250	300	300	300	300	300	300	300	300	300
26	7	38	250	200	200	200	200	200	200	200	200	200
27	7	27	200	250	250	250	250	250	250	250	250	250
28	8	48	500	300	300	300	300	300	300	300	300	300
29	8	50	500	200	200	200	200	200	200	200	200	200
30	8	57	300	150	150	150	150	150	150	150	150	150
31	9	76	500	300	300	300	300	300	300	300	300	300
32	9	60	500	200	200	200	200	200	200	200	200	200
33	9	62	300	150	150	150	150	150	150	150	150	150
34	10	26	100	150	150	150	150	150	150	150	150	150
35	10	11	250	300	300	300	300	300	300	300	300	300
36	10	38	250	200	200	200	200	200	200	200	200	200
37	10	27	200	250	250	250	250	250	250	250	250	250
38	11	48	500	300	300	300	300	300	300	300	300	300
39	11	50	500	200	200	200	200	200	200	200	200	200
40	11	46	300	150	150	150	150	150	150	150	150	150
41	12	72	300	150	150	150	150	150	150	150	150	150
42	13	100	300	150	150	150	150	150	150	150	150	150
43	14	84	200	250	250	250	250	250	250	250	250	250
44	15	7	200	250	250	250	250	250	250	250	250	250
45	16	67	300	150	150	150	150	150	150	150	150	150
46	17	100	300	150	150	150	150	150	150	150	150	150
47	18	47	200	250	250	250	250	250	250	250	250	250
48	19	28	200	250	250	250	250	250	250	250	250	250
49	20	80	300	150	150	150	150	150	150	150	150	150
50	21	45	200	250	250	250	250	250	250	250	250	250
51	22	49	150	1000	500	0	3000	100	2200	500	250	500
52	22	38	100	200	0	100	500	0	0	0	0	100
53	22	78	100	500	150	0	3500	1800	0	2500	75	250
54	22	50	0	0	0	0	100	1500	0	1000	0	0
55	22	48	500	2000	750	500	1500	350	800	150	375	1000
56	23	62	300	1000	1000	0	6000	200	2200	1000	500	500
57	23	60	200	200	0	200	1000	0	0	0	0	100
58	23	61	200	500	300	0	7000	3600	0	5000	150	250
59	23	63	0	0	0	0	200	3000	0	2000	0	0
60	23	65	1000	2000	1500	1000	3000	700	800	300	750	1000
61	24	61	500	300	300	300	300	300	300	300	300	300
62	24	60	500	200	200	200	200	200	200	200	200	200
63	24	62	300	150	150	150	150	150	150	150	150	150
64	25	1	100	150	150	150	150	150	150	150	150	150
65	25	11	250	300	300	300	300	300	300	300	300	300
66	25	38	250	200	200	200	200	200	200	200	200	200
67	25	27	200	250	250	250	250	250	250	250	250	250
68	26	48	500	300	300	300	300	300	300	300	300	300
69	26	50	500	200	200	200	200	200	200	200	200	200

70	26	49	300	150	150	150	150	150	150	150	150	150
71	27	59	500	300	300	300	300	300	300	300	300	300
72	27	60	500	200	200	200	200	200	200	200	200	200
73	27	62	300	150	150	150	150	150	150	150	150	150
74	28	4	100	150	150	150	150	150	150	150	150	150
75	28	11	250	300	300	300	300	300	300	300	300	300
76	28	38	250	200	200	200	200	200	200	200	200	200
77	28	27	200	250	250	250	250	250	250	250	250	250
78	29	48	500	300	300	300	300	300	300	300	300	300
79	29	50	500	200	200	200	200	200	200	200	200	200
80	29	86	300	150	150	150	150	150	150	150	150	150
81	30	72	500	300	300	300	300	300	300	300	300	300
82	30	60	500	200	200	200	200	200	200	200	200	200
83	30	62	300	150	150	150	150	150	150	150	150	150
84	31	35	100	150	150	150	150	150	150	150	150	150
85	31	11	250	300	300	300	300	300	300	300	300	300
86	31	38	250	200	200	200	200	200	200	200	200	200
87	31	27	200	250	250	250	250	250	250	250	250	250
88	32	48	500	300	300	300	300	300	300	300	300	300
89	32	50	500	200	200	200	200	200	200	200	200	200
90	32	87	300	150	150	150	150	150	150	150	150	150
91	33	42	200	250	250	250	250	250	250	250	250	250
92	34	36	200	250	250	250	250	250	250	250	250	250
93	35	43	200	250	250	250	250	250	250	250	250	250
94	36	15	200	250	250	250	250	250	250	250	250	250
95	37	52	300	150	150	150	150	150	150	150	150	150
96	38	59	300	150	150	150	150	150	150	150	150	150
97	39	98	200	250	250	250	250	250	250	250	250	250
98	40	54	300	150	150	150	150	150	150	150	150	150
99	41	90	300	150	150	150	150	150	150	150	150	150
100	42	34	200	250	250	250	250	250	250	250	250	250

Figure B5. SSC Demand Set

The demand set for the Mix scenario is provided in Figure B6.

Dem #	Con #	Loc #	Weapon # 1	2	3	4	5	6	7	8	9	10
1	1	49	150	1000	500	0	3000	100	2200	500	250	500
2	1	38	100	200	0	100	500	0	0	0	0	100
3	1	78	100	500	150	0	3500	1800	0	2500	75	250
4	1	50	0	0	0	0	100	1500	0	1000	0	0
5	1	48	500	2000	750	500	1500	350	800	150	375	1000
6	2	62	300	1000	1000	0	6000	200	2200	1000	500	500
7	2	60	200	200	0	200	1000	0	0	0	0	100
8	2	61	200	500	300	0	7000	3600	0	5000	150	250
9	2	63	0	0	0	0	200	3000	0	2000	0	0
10	2	65	1000	2000	1500	1000	3000	700	800	300	750	1000
11	3	55	150	1000	500	0	3000	100	2200	500	250	500
12	3	38	100	200	0	100	500	0	0	0	0	100
13	3	78	100	500	150	0	3500	1800	0	2500	75	250
14	3	50	0	0	0	0	100	1500	0	1000	0	0
15	3	48	500	2000	750	500	1500	350	800	150	375	1000
16	4	62	300	1000	1000	0	6000	200	2200	1000	500	500
17	4	60	200	200	0	200	1000	0	0	0	0	100
18	4	61	200	500	300	0	7000	3600	0	5000	150	250
19	4	63	0	0	0	0	200	3000	0	2000	0	0
20	4	64	1000	2000	1500	1000	3000	700	800	300	750	1000
21	5	61	500	300	300	300	300	300	300	300	300	300
22	5	60	500	200	200	200	200	200	200	200	200	200
23	5	62	300	150	150	150	150	150	150	150	150	150
24	6	1	100	150	150	150	150	150	150	150	150	150
25	6	11	250	300	300	300	300	300	300	300	300	300
26	6	38	250	200	200	200	200	200	200	200	200	200
27	6	27	200	250	250	250	250	250	250	250	250	250
28	7	48	500	300	300	300	300	300	300	300	300	300
29	7	50	500	200	200	200	200	200	200	200	200	200
30	7	49	300	150	150	150	150	150	150	150	150	150
31	8	61	500	300	300	300	300	300	300	300	300	300
32	8	60	500	200	200	200	200	200	200	200	200	200
33	8	69	300	150	150	150	150	150	150	150	150	150
34	9	5	100	150	150	150	150	150	150	150	150	150
35	9	11	250	300	300	300	300	300	300	300	300	300
36	9	38	250	200	200	200	200	200	200	200	200	200
37	9	27	200	250	250	250	250	250	250	250	250	250
38	10	48	500	300	300	300	300	300	300	300	300	300
39	10	50	500	200	200	200	200	200	200	200	200	200
40	10	56	300	150	150	150	150	150	150	150	150	150
41	11	6	200	250	250	250	250	250	250	250	250	250

42	12	75	300	150	150	150	150	150	150	150	150	150
43	13	12	200	250	250	250	250	250	250	250	250	250
44	14	16	200	250	250	250	250	250	250	250	250	250
45	15	18	200	250	250	250	250	250	250	250	250	250
46	16	79	300	150	150	150	150	150	150	150	150	150
47	17	88	200	250	250	250	250	250	250	250	250	250
48	18	20	200	250	250	250	250	250	250	250	250	250
49	19	82	300	150	150	150	150	150	150	150	150	150
50	20	41	200	250	250	250	250	250	250	250	250	250
51	21	49	150	1000	500	0	3000	100	2200	500	250	500
52	21	38	100	200	0	100	500	0	0	0	0	100
53	21	78	100	500	150	0	3500	1800	0	2500	75	250
54	21	50	0	0	0	0	100	1500	0	1000	0	0
55	21	48	500	2000	750	500	1500	350	800	150	375	1000
56	22	62	300	1000	1000	0	6000	200	2200	1000	500	500
57	22	60	200	200	0	200	1000	0	0	0	0	100
58	22	61	200	500	300	0	7000	3600	0	5000	150	250
59	22	63	0	0	0	0	200	3000	0	2000	0	0
60	22	65	1000	2000	1500	1000	3000	700	800	300	750	1000
61	23	53	150	1000	500	0	3000	100	2200	500	250	500
62	23	38	100	200	0	100	500	0	0	0	0	100
63	23	78	100	500	150	0	3500	1800	0	2500	75	250
64	23	50	0	0	0	0	100	1500	0	1000	0	0
65	23	48	500	2000	750	500	1500	350	800	150	375	1000
66	24	62	300	1000	1000	0	6000	200	2200	1000	500	500
67	24	60	200	200	0	200	1000	0	0	0	0	100
68	24	61	200	500	300	0	7000	3600	0	5000	150	250
69	24	63	0	0	0	0	200	3000	0	2000	0	0
70	24	70	1000	2000	1500	1000	3000	700	800	300	750	1000
71	25	61	500	300	300	300	300	300	300	300	300	300
72	25	60	500	200	200	200	200	200	200	200	200	200
73	25	62	300	150	150	150	150	150	150	150	150	150
74	26	1	100	150	150	150	150	150	150	150	150	150
75	26	11	250	300	300	300	300	300	300	300	300	300
76	26	38	250	200	200	200	200	200	200	200	200	200
77	26	27	200	250	250	250	250	250	250	250	250	250
78	27	48	500	300	300	300	300	300	300	300	300	300
79	27	50	500	200	200	200	200	200	200	200	200	200
80	27	49	300	150	150	150	150	150	150	150	150	150
81	28	61	500	300	300	300	300	300	300	300	300	300
82	28	60	500	200	200	200	200	200	200	200	200	200
83	28	76	300	150	150	150	150	150	150	150	150	150
84	29	13	100	150	150	150	150	150	150	150	150	150
85	29	11	250	300	300	300	300	300	300	300	300	300
86	29	38	250	200	200	200	200	200	200	200	200	200
87	29	27	200	250	250	250	250	250	250	250	250	250
88	30	48	500	300	300	300	300	300	300	300	300	300

89	30	50	500	200	200	200	200	200	200	200	200	200
90	30	51	300	150	150	150	150	150	150	150	150	150
91	31	97	300	150	150	150	150	150	150	150	150	150
92	32	85	300	150	150	150	150	150	150	150	150	150
93	33	30	300	150	150	150	150	150	150	150	150	150
94	34	96	300	150	150	150	150	150	150	150	150	150
95	35	74	300	150	150	150	150	150	150	150	150	150
96	36	97	300	150	150	150	150	150	150	150	150	150
97	37	57	300	150	150	150	150	150	150	150	150	150
98	38	54	300	150	150	150	150	150	150	150	150	150
99	39	85	300	150	150	150	150	150	150	150	150	150
100	40	21	200	250	250	250	250	250	250	250	250	250

Figure B6. Mix Demand Set

Appendix C

Efficient Frontiers

The following figures represent the multiple objective efficient frontiers found during the experimental phase of the study while using equal weighting of objectives.

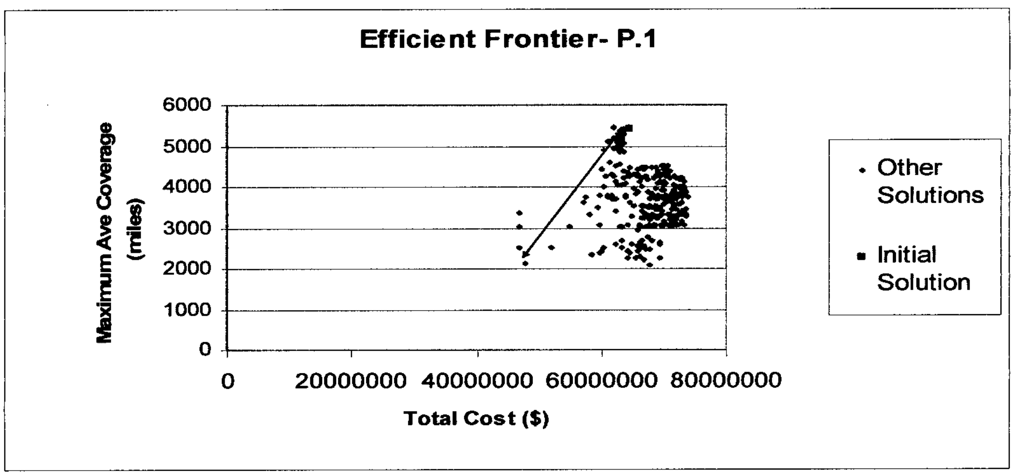

Figure C1. Efficient Frontier – Problem 1

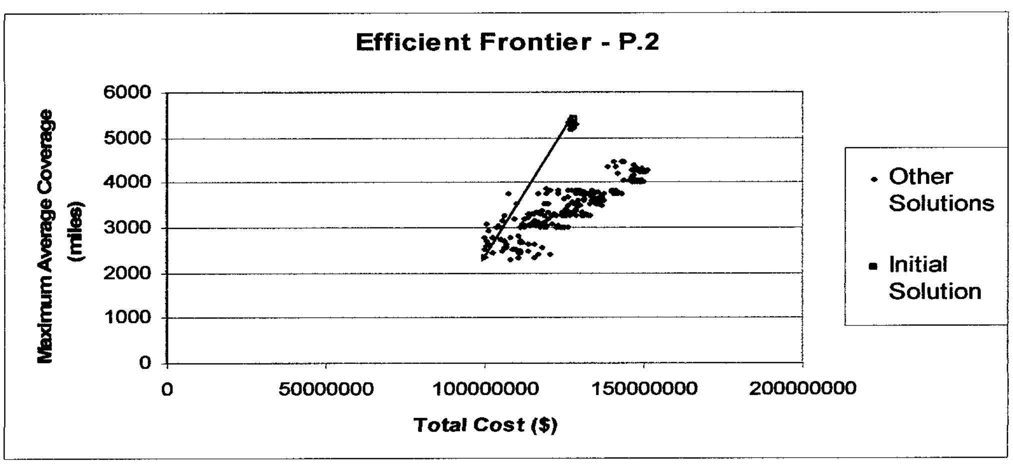

Figure C2. Efficient Frontier – Problem 2

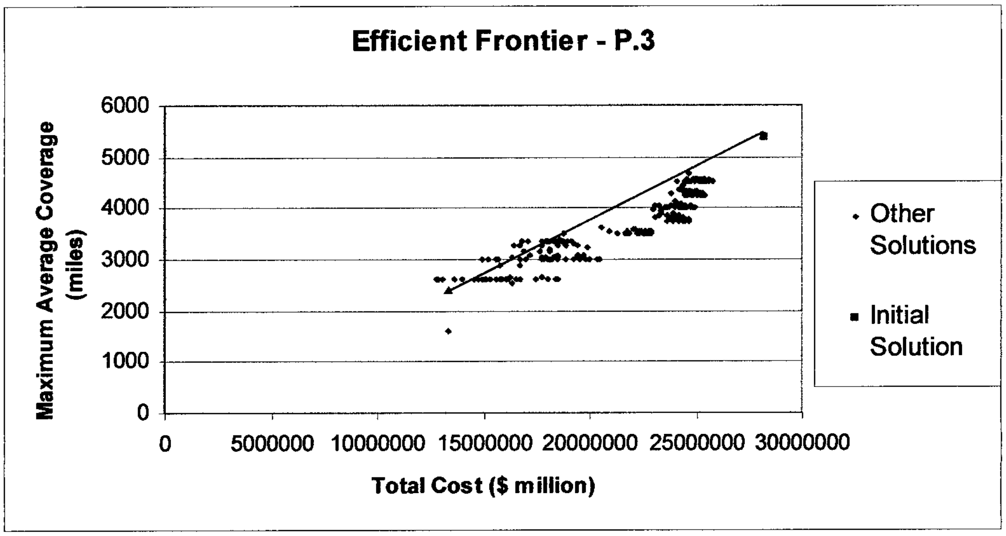

Figure C3. Efficient Frontier – Problem 3

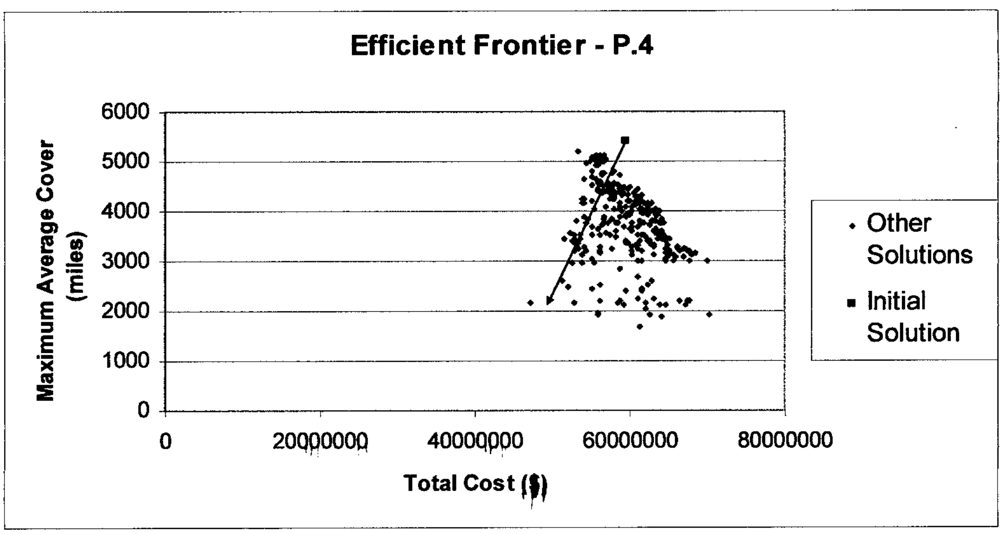

Figure C4. Efficient Frontier – Problem 4

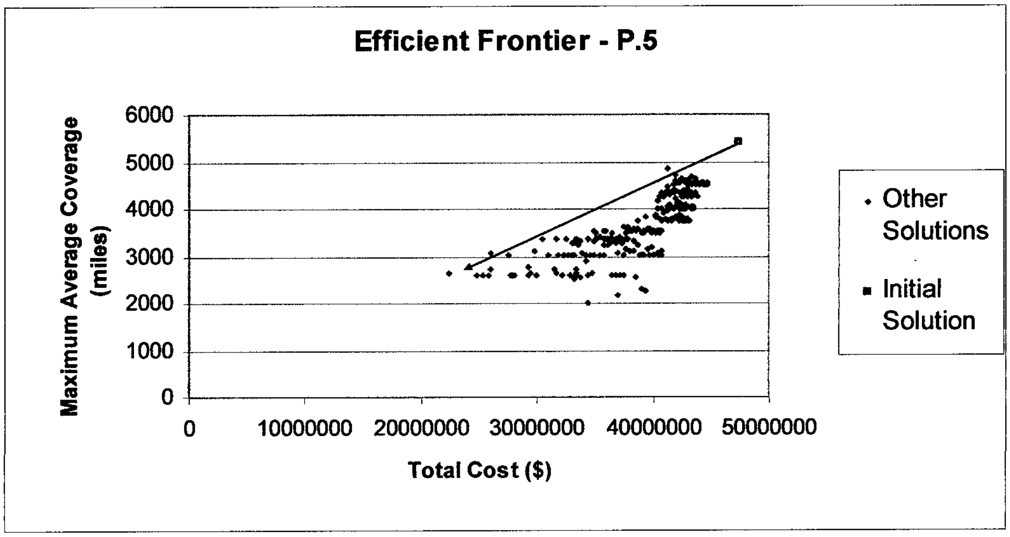

Figure C5. Efficient Frontier – Problem 5

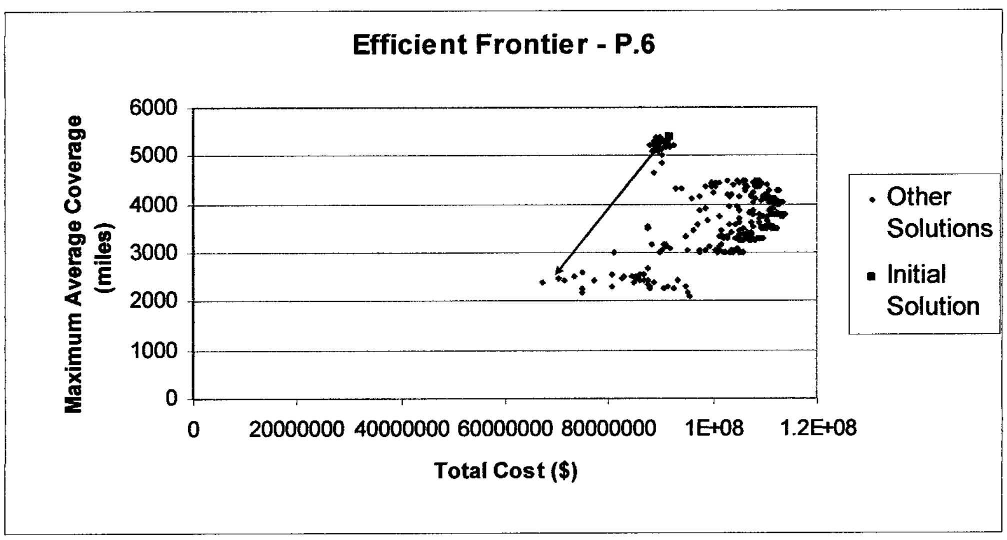

Figure C6. Efficient Frontier – Problem 6